Optimal Government Spending in a Collateral-Constrained Small Open Economy

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Abstract

This paper investigates the stabilization role of government spending in a collateral-constrained small open economy. The economy is characterized by inefficiencies in borrowing decisions, resulting from pecuniary externalities and the amplification mechanism of the debt-deflation spiral. In this context, government spending serves to maintain financial stability, extending beyond the efficient provision of public goods. When the economy borrows up to its limit, the optimal response is fiscal stimulus, which mitigates the amplification of the debt-deflation mechanism. The optimal time-consistent policy prevents recessionary shocks from leading to a financial crisis accompanied by a drastic reversal of the current account. We show that an implementable government spending policy, which maintains a constant ratio to GDP, approximates the optimal policy and achieves a second-best outcome.

Keywords: Small open economy; financial crises; optimal government spending.

JEL classification: F41, F44, E44, G01

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1 Introduction

A number of emerging economies have experienced significant reversals in their current account balances. Extensive literature has explored the root causes of these sudden stops, attributing them to the presence of collateral constraints that lead to inefficiencies arising from pecuniary externalities and the amplification mechanism of the debt-deflation spiral. Various policy interventions have been proposed by Mendoza (2010), Bianchi (2011), Benigno et al. (2016), Schmitt-Grohé and Uribe (2017), and Bianchi and Mendoza (2018) among others. The pecuniary externality emerges because individual borrowers do not consider the impact of their decisions on collateral prices, even though these prices are endogenously determined in the economy based on aggregate demand. Consequently, borrowing decisions influenced by this pecuniary externality lead to overborrowing, rendering the economy financially vulnerable.[1]

Capital control policies have been investigated as a means to correct pecuniary externalities by Bianchi (2011), Schmitt-Grohé and Uribe (2017), and Bianchi and Mendoza (2018). It has been demonstrated that capital controls enable households to internalize the unconsidered effects of their borrowing decisions on collateral prices, thereby reducing the economy's vulnerability.

As an ex-post policy intervention, Benigno et al. (2016) have examined collateral price support policies. This type of policy is not aimed at correcting the pecuniary externality but at preventing the economy from entering a debt-deflation spiral by appreciating the collateral price ex post. The debt-deflation spiral occurs when the collateral constraint becomes binding. The reduction in capital inflows following the binding of the constraint leads to decreased domestic demand and a deflation in collateral prices. This, in turn, diminishes the value of collateral, making further borrowing more challenging. The ex-post intervention supports the collateral price, thereby mitigating the debt-deflation spiral.

There is a prevailing view that government spending should serve as a stabilization tool in business cycles, particularly when monetary policy is constrained. In small open

[1] In specific instances, underborrowing has also been identified by Benigno et al. (2013), Chi et al. (2021) and Schmitt-Grohé and Uribe (2021).

economies susceptible to sudden stops, monetary policy is limited in its capacity to stabilize prices and employment due to the need to also consider financial stability (Ottonello, 2021 and Coulibaly, 2023). Additionally, economies in a currency union, such as peripheral European small open economies that experienced a sudden stop of capital inflows during the global financial crisis, lack autonomy in monetary policy.[2] These considerations provide a rationale for the role of government spending, especially when the efficacy of monetary policy is restricted. While the stabilization role of government spending with nominal rigidities has been well studied (for example, Bianchi et al., 2023), the stabilization role of fiscal spending against frictions arising from the collateral constraint and pecuniary externality has not yet been thoroughly explored. This paper aims to address this gap.

To achieve this, we characterize the optimal government spending policy in collateral-constrained small open economies. In our model, households' borrowing capacity is limited to a fraction of their current income, which consists of endowment receipts from tradable and nontradable goods. With tradable goods as the numeraire, the crucial collateral price is the relative price of nontradable goods. Because households do not consider the effects of their consumption choices on the relative price, a pecuniary externality arises. As households undervalue the marginal utility of wealth compared to the socially optimal level, they accumulate an inefficiently high level of borrowing. Furthermore, during economic downturns, the value of collateral depreciates due to the sudden stop of capital inflows and contractions in domestic absorption, triggering a debt-deflation spiral.

In this context, fiscal spending plays a stabilizing role in business cycles. In our model, where households derive direct utility from public consumption, the efficient provision of public goods is achieved when the marginal direct utility of public consumption equals the marginal cost of crowded-out private consumption, as per Samuelson (1954), i.e., the Samuelson rule. In a collateral-constrained economy, the role of government spending extends beyond the efficient provision of public goods to include maintaining financial

[2]Gali and Monacelli (2008) underscore the stabilization role of government spending in small open economies with nominal rigidities that are part of a currency union.

stability. Ex ante, government spending policy can help mitigate the overborrowing problem. When the collateral constraint is not binding, under plausible parameter values, fiscal austerity discourages overborrowing due to pecuniary externality today by altering the intertemporal allocation to the efficient level and hence reducing the probability of a future crisis. Conversely, government spending can be used to sustain the level of capital inflows ex-post. When the collateral constraint binds, the appreciation of collateral prices leads to increased borrowing, as the borrowing level is determined by the value of collateral. Government spending on nontradable goods appreciates the price of collateral. Thus, when the constraint binds, fiscal stimulus enables the economy to maintain the level of capital inflows even when borrowing reaches its upper limit. This prevents the economy from falling into a vicious debt-deflation spiral. The optimal time-consistent government spending balances the efficient provision of public goods with financial stability.

Quantitative analyses suggest that deviating from the Samuelson rule, which prioritizes the efficient provision of public goods while neglecting financial stability, plays a crucial role in stabilizing the economy when the collateral constraint is binding. The optimal fiscal policy, which incorporates significant welfare gains from ex-post fiscal stimulus, outperforms the Samuelson rule. According to our simulations, the penalty associated with the binding constraint, as indicated by the average shadow value of relaxing the collateral constraint, is 0.005 under the optimal policy and 0.016 under the Samuelson rule. The ex-post stimulus sustains capital inflows even when borrowing reaches its upper limit and alleviates intertemporal distortions caused by the borrowing constraint. Additionally, it leads to higher borrowing, as the diminished likelihood of the collateral constraint being binding reduces the households' precautionary saving motive. The ex-post fiscal stimulus results in a 0.23 percent welfare gain in permanent total consumption.

However, adhering to the Samuelson rule when the collateral constraint is not binding also contributes to financial stability, even if it is not explicitly intended. The reason is that any level of government spending appreciates the price of collateral to some extent and allows a higher level of capital inflows without reaching the borrowing limit. The point is hilighted by comparing the outcome under the Samuelson rule and a zero-spending policy. The average shadow value of relaxing the collateral constraint is 0.016 under the Samuelson rule, compared to 0.017 without any fiscal intervention. Numerical analyses illustrate that the optimal policy does not deviate much from the Samuelson rule when the collateral constraint is not binding, because the gain from maintaining the financial stability through a higher collateral value outperforms the gain from reducing overborrowing by ex-ante fiscal austerity.

We demonstrate that the optimal fiscal policy, which balances the efficient provision of public goods and financial stability, significantly reduces the frequency and severity of financial crises and drastic current account reversals. The probability of financial crises is 0.0005 percent under the optimal policy, compared to 5.26 percent without any policy intervention. Recognizing the challenge of implementing the optimal policy, which necessitates detailed information about the economy, we propose that a government spending rule that maintains a constant spending-to-GDP ratio effectively approximates the optimal policy in our environment.

Our paper contributes to the recent literature on government spending in small open economies. Liu (2022) explores the transmission mechanism of government spending in a collateral-constrained small open economy, showing that during sudden stop crises, the government spending multiplier on private consumption is higher compared to normal times. This is attributed to fiscal expansion appreciating the value of collateral and enabling more borrowing during crises. While our paper shares this transmission mechanism, we further characterize the optimal government spending policy, which is not addressed in the previous research.

Bianchi et al. (2023) examine the optimal fiscal policy during a recession in a small open economy with downward nominal wage rigidity and endogenous sovereign defaults. They demonstrate quantitatively that the stimulus benefits of reducing unemployment during the recession are offset by the risk of debt crises, making fiscal expansion potentially undesirable. In contrast, our paper focuses on the optimal spending under pecuniary externalities and the debt-deflation spiral, rather than nominal rigidities and sovereign default risks.

More broadly, our paper is related to the literature on policy intervention in collateral-constrained small open economies. Devereux et al. (2019) and Matsumoto (2021) investigate monetary and macroprudential policies in such economies. Davis et al. (2023) examine foreign reserve policies to prevent sudden stop crises. Chi et al. (2021) consider the impact of interest rate policies on central bank reserves to prevent household deleveraging from leading to aggregate deleveraging. Korinek and Sandri (2016) differentiate between capital controls in foreign lending and domestic macroprudential regulation. Benigno et al. (2023) identify a set of policy instruments, including those that can manipulate the price of collateral, which can implement the constrained efficient allocation and restore the allocation without the collateral constraint in the model of Bianchi (2011) and Benigno et al. (2013). Durdu and Mendoza (2006) investigate asset price guarantees.

The remainder of the paper is organized as follows. Section 2 presents our small open economy model. Section 3 characterizes the optimal government spending, and Section 4 conducts a quantitative analysis of the model's characteristics and the optimal government spending. Section 5 concludes the paper.

2 The Model

In this section, we introduce our model environment. The model is based on a prototypical small open economy model of Bianchi (2011). We introduce government consumption from which households gain the utility.[3] The representative households maximize the lifetime expected utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u\left(c_t\right) + v\left(g_t\right) \right\} \tag{1}$$

where c_t denotes consumption in period t and g_t denotes the government spending in nontradable goods in period t which we assume provides direct utility. $\beta \in (0,1)$ is a subjective discount factor. We assume $u\left(c_t\right) = \frac{c_t^{1-\sigma}}{1-\sigma}$ and $v\left(g_t\right) = \chi \frac{g_t^{1-\sigma}}{1-\sigma}$ where $\sigma > 0$ is the

[3]The assumption of the direct utility from public spending is common in the literature on optimal government spending both in open and closed economies, for example, Gali and Monacelli (2008), Nakata (2016), Bilbiie et al. (2019) and Bianchi et al. (2023) among many others.

inverse of the intertemporal elasticity of substitution. We assume the same degree of risk aversion for private and public consumption as Bianchi et al. (2023). The consumption basket is a composite of tradable and nontradable goods and is given by a CES aggregator,

$$c_t = \left[a\left(c_t^T\right)^{1-\frac{1}{\xi}} + (1-a)\left(c_t^N\right)^{1-\frac{1}{\xi}}\right]^{\frac{1}{1-\frac{1}{\xi}}} \tag{2}$$

where c_t^T denotes consumption of tradable goods in period t and c_t^N denotes consumption of nontradable goods in period t. The parameter $a \in (0,1)$ governs the weight of tradable goods in the consumption basket and $\xi > 0$ denotes the intratemporal elasticity of substitution between tradable and nontradable consumption. In every period t, households receive the endowment of tradable and nontradable goods, denotes by y_t^T and y_t^N, respectively. Both endowments are exogenously given. We assume that households have access to one period, non-state contingent, internationally traded bond denominated in terms of tradable goods. Holding this bond from period t to period $t+1$ pays the interest rate r. In addition, households pay lump-sum tax T_t in nontradable goods which is used to finance government spending. The household's period-by-period budget constraint is given by

$$c_t^T + p_t c_t^N + d_t = y_t^T + p_t y_t^N + \frac{d_{t+1}}{1+r} - p_t T_t \tag{3}$$

where d_{t+1} denotes the amount of bond assumed in period t and due in period $t+1$ and p_t is the relative price of nontradable goods in terms of tradable goods.[4] In addition to the sequential budget constraint, households face the collateral constraint of the form

$$d_{t+1} \leq \kappa\left(y_t^T + p_t y_t^N\right) \tag{4}$$

where $\kappa > 0$ is the parameter governing the tightness of the collateral constraint. This constraint states that a household's borrowing in each period is restricted to be less than or equal to some fraction of the value of current income. The externality arises because each

[4]The real exchange rate is represented by $RER_t \frac{\left[a^{*\frac{1}{\sigma^*}} + (1-a^*)^{\frac{1}{\sigma^*}}(p_t^*)^{\frac{\sigma^*-1}{\sigma^*}}\right]^{\frac{\sigma^*}{\sigma^*-1}}}{\left[a^{\frac{1}{\sigma}} + (1-a)^{\frac{1}{\sigma}}(p_t)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}}$ in the current economy assuming the law of one price for tradable goods, where asterisk indicates parameters and a variable in the rest of the world. Since p_t^* is exogenous for the small open economy, we will refer to the relative price and the real exchange rate interchangeably.

household takes as given the value of collateral, which is the relative price of nontradables, even though their aggregate choice of tradable and nontradable consumption determines this price.

Households maximize (1) subject to (2), (3) and (4) by choosing c_t^T, c_t^N, c_t and d_{t+1}. The first order conditions are (2), (3) , (4),

$$c_t^{-\sigma} c_t^{\frac{1}{\xi}} a \left(c_t^T\right)^{-\frac{1}{\xi}} = \lambda_t, \tag{5}$$

$$p_t = \frac{1-a}{a} \left(\frac{c_t^T}{c_t^N}\right)^{\frac{1}{\xi}}, \tag{6}$$

$$\frac{\lambda_t}{1+r} - \mu_t = \beta E_t \lambda_{t+1}, \tag{7}$$

$$\mu_t \left\{\kappa \left(y_t^T + p_t y_t^N\right) - d_{t+1}\right\} = 0, \tag{8}$$

and

$$\mu_t \geq 0,$$

where λ_t denotes the Lagrange multiplier on the sequential budget constraint and μ_t is the Lagrange multiplier on the collateral constraint. The Euler equation (7) equates the marginal value of an additional unit of borrowing and its marginal cost. When the collateral constraint is not binding the former is $\frac{\lambda_t}{1+r}$ and the latter is $\beta E_t \lambda_{t+1}$, and the marginal rate of substitution is equal to $1+r$. The binding of collateral constraint adds the wedge into this relation with a positive value of relaxing the collateral constraint, $\mu_t > 0$, falling the marginal utility of additional borrowing. The marginal rate of substitution is no longer equal to $1+r$.

We assume that government spending on nontradable goods is financed by lump-sum taxation without adding any distortions. The government's sequential budget constraint is given by

$$g_t = T_t.$$

Combining sequential private and government budget constraints and nontradable goods market clearing condition,

$$c_t^N + g_t = y_t^N \tag{9}$$

yields the resource constraint of the economy given by

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1+r}. \tag{10}$$

The increase in government spending appreciates the relative price of nontradable goods as is obvious by inserting nontradable goods market clearing condition (9) into (6).[5] In the current small open economy changes in the relative price do not affect the allocation of the tradable goods when the collateral constraint is not binding. This is because the Euler equation and the resource constraint determine the amount of borrowing and tradable consumption during the slack of collateral constraint. However, since the value of collateral is manipulated, the level of indebtedness at which the collateral constraint binds changes depending on the government spending. In contrast, when the constraint is binding, changes in the price of the collateral directly determine the amount of borrowing and tradable consumption. Specifically, even when the constraint holds with equality, higher government spending enables further borrowing by appreciating the price of the collateral. In addition, irrespective of the condition of external financing, government spending changes the marginal utility of borrowing unless the intra- and intertemporal elasticity of substitution are equal. In the case where the intratemporal elasticity of substitution is larger than the intertemporal elasticity of substitution ($\xi > \frac{1}{\sigma}$) as in our numerical exercise below, the marginal utility of borrowing is increasing in government spending. With other things constant, the temporary fiscal stimulus encourages the borrowing. In contrast, when the intratemporal elasticity is lower than the intertemporal elasticity ($\xi < \frac{1}{\sigma}$), the marginal utility of borrowing is decreasing in government spending. In this case, the temporary fiscal stimulus discourages the borrowing.

[5]The appreciation of the real exchange rate following the fiscal stimulus is empirically observed in emerging economies. See Miyamoto et al. (2019).

The competitive equilibrium is a set of processes $\left\{c_t^T, c_t, d_{t+1}, \lambda_t, \mu_t\right\}$ satisfying

$$c_t = \left[a\left(c_t^T\right)^{1-\frac{1}{\xi}} + (1-a)\left(y_t^N - g_t\right)^{1-\frac{1}{\xi}}\right]^{\frac{1}{1-\frac{1}{\xi}}}, \tag{11}$$

$$c_t^{-\sigma} c_t^{\frac{1}{\xi}} a \left(c_t^T\right)^{-\frac{1}{\xi}} = \lambda_t, \tag{12}$$

$$\frac{\lambda_t}{1+r} - \mu_t = \beta E_t \lambda_{t+1}, \tag{13}$$

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1+r}, \tag{14}$$

$$d_{t+1} \leq \kappa \left(y_t^T + \frac{1-a}{a}\left(\frac{c_t^T}{y_t^N - g_t}\right)^{\frac{1}{\xi}} y_t^N\right), \tag{15}$$

$$\mu_t \left\{\kappa \left(y_t^T + \frac{1-a}{a}\left(\frac{c_t^T}{y_t^N - g_t}\right)^{\frac{1}{\xi}} y_t^N\right) - d_{t+1}\right\} = 0, \tag{16}$$

and

$$\mu_t \geq 0, \tag{17}$$

given a process $\{g_t\}$ and exogenous processes $\left\{y_t^T, y_t^N\right\}$. As we can see by substituting (14) into the right-hand side of (15), the value of the collateral is increasing in d_{t+1} and it may increase more than one for one with d_{t+1} depending on parameter values. As discussed by Schmitt-Grohé and Uribe (2021), due to this feature, the possibility multiple equilibria arises. However, in the quantitative analysis, we adopt parameterization under which such a possibility does not arise.

3 Optimal Government Spending

In this section, we characterize the optimal government spending. The government maximizes a household's utility by employing government spending as a sole policy instrument subject competitive equilibrium conditions. Since the constraint set includes Euler equation of households, the problem of the government is dynamic. Thus, we assume that the government lacks the commitment device and look for a Markov-perfect equilibrium as in Bianchi and Mendoza (2018), Devereux et al. (2019), and Coulibaly (2023). The current government takes as given the future government's decisions, but considers the effects of her choice of borrowing, which is the state variable of the next period, on those decisions. Let $G\left(d, y^T, y^N\right)$ be the government spending choice of future government that current government takes as given and $C^T\left(d, y^T, y^N\right)$ the associated functions that give the values of tradable goods consumption under that policy. Given these functions, the problem of government is described as

$$V\left(d, y^T, y^N\right) = \max_{c^T, d', \mu, g} u\left[c\left(c^T, y^N - g\right)\right] + v\left(g\right) + \beta E_{y^{T\prime}, y^{N\prime} | y^T, y^N} V\left(d', y^{T\prime}, y^{N\prime}\right) \tag{18}$$

subject to

$$c = \left[a\left(c^T\right)^{1-\frac{1}{\xi}} + (1-a)\left(y^N - g\right)^{1-\frac{1}{\xi}}\right]^{\frac{1}{1-\frac{1}{\xi}}}, \tag{19}$$

$$c^T + d = y^T + \frac{d'}{1+r}, \tag{20}$$

$$\frac{u_T\left(c^T, y^N - g\right)}{1+r} - \mu = \beta E_{y^{T\prime}, y^{N\prime} | y^T, y^N}\left[u_T\left(C^T\left(d', y^{T\prime}, y^{N\prime}\right), y^{N\prime} - G\left(d', y^{T\prime}, y^{N\prime}\right)\right)\right], \tag{21}$$

$$\kappa\left[y^T + \frac{1-a}{a}\left(\frac{c^T}{y^N - g}\right)^{\frac{1}{\xi}} y^N\right] \geq d', \tag{22}$$

$$\mu\left[\kappa\left(y^T + \frac{1-a}{a}\left(\frac{c^T}{y^N - g}\right)^{\frac{1}{\xi}} y^N\right) - d'\right] = 0, \tag{23}$$

and

$$\mu \geq 0, \tag{24}$$

where $u_T \equiv \frac{\partial u\left(c^T, y^N - g\right)}{\partial c^T}$ is the marginal utility of tradable consumption and a prime superscript denotes the variable in the next period.

We examine the first order condition with respect to the government spending in the government problem. It is given by

$$\frac{\partial u\left(c^T, y^N - g\right)}{\partial g} + \frac{\partial v\left(g\right)}{\partial g} + \left(\lambda^3 + \lambda^4 \mu\right) \kappa \frac{\partial p}{\partial g} y^N + \frac{\lambda^2}{1+r} \frac{\partial u_T\left(c^T, y^N - g\right)}{\partial g} = 0, \tag{25}$$

where λ^2, λ^3 and λ^4 are the multipliers associated with the private Euler equation (21), the collateral constraint (22) and the slackness condition (23). The first two terms of (25) balance the marginal decline in the utility due to crowded-out nontradable consumption and the marginal increase in the direct utility from government spending. The efficient provision of public goods is attained when the level of spending is determined so that these terms are equated. Facing the collateral constraint, the government may find it optimal to deviate from the efficient provision of public goods to maintain financial stability, as expressed by third and fourth terms of (25). Noting that the relative price of nontradable is increasing in government spending, fiscal stimulus yields benefits because it allows to continue to borrow even when the economy hits the upper limit of borrowing. The maintained level of capital inflows helps the economy with escaping from the debt-deflation spiral. The third term represents this benefit. The marginal increase in government spending increases the value of collateral by the size of $\kappa \frac{\partial p}{\partial g} y^N$. Noting that $\lambda^3 + \lambda^4 \mu$ is the government's effective shadow value of relaxing the collateral constraint which is positive when the constraint binds, the ex post fiscal stimulus has the utility benefits of $\left(\lambda^3 + \lambda^4 \mu\right) \kappa \frac{\partial p}{\partial g} y^N$. In contrast, the fourth term of (25) captures the utility benefit of ex ante intervention. This term appears in the case where the collateral constraint does not bind and government spending affects the marginal utility of tradable consumption.[6] As discussed by Bianchi (2011) and among others, there is a wedge between private and socially optimal value of borrowing arising from the pecuniary externality. This wedge results in the inefficiently high level of borrowing when the collateral constraint is not binding. The government recognizes this ex ante inefficiency and mitigates the overborrowing through changing the marginal utility of borrowing while the collateral constraint is slack. As discussed above, when the intratemporal elasticity of substitution

[6]The multiplier on the private Euler equation is zero ($\lambda^2 = 0$) when the collateral constraint binds. See Appendix A.

is greater (less) than the intertemporal elasticity of substitution, the marginal utility of borrowing is increasing (decreasing) in government spending. Thus, the current fiscal austerity (expansion) improves the financial stability.

4 Numerical Exercises

In this section, we investigate the quantitative property of the model.

4.1 Calibration

Our calibration of the model follows exactly the one in Bianchi (2011). The parameter values are summarized in Table 1. The model is calibrated on an annual basis. The value of parameter $\kappa = 0.32\,(1+r)$ is different from $\kappa = 0.32$ in Bianchi (2011), because our specification of collateral constraint, $d_{t+1} \leq \kappa\left(y_t^T + p_t y_t^N\right)$, which is the same as Schmitt-Grohé and Uribe (2017) and Schmitt-Grohé and Uribe (2021), is different from $\frac{d_{t+1}}{1+r} \leq \kappa\left(y_t^T + p_t y_t^N\right)$ in Bianchi (2011). Our resulting calibration is the same as the one in Bianchi (2011), as discussed in Schmitt-Grohé and Uribe (2017). The weight of the utility from public spending is set to match the average government spending to GDP ratio in Argentina which is 10 percent during the periods from 1965 to 2007. The natural logarithms of tradable and nontradable endowments follow a bivariate AR(1) process. We borrow the estimates of this process by Bianchi (2011) who estimates with the HP-filtered Argentine data for the periods from 1965 to 2007. The estimated process is given by

$$\begin{bmatrix} \ln y_t^T \\ \ln y_t^N \end{bmatrix} = \begin{bmatrix} 0.901 & 0.495 \\ -0.453 & 0.225 \end{bmatrix} \begin{bmatrix} \ln y_{t-1}^T \\ \ln y_{t-1}^N \end{bmatrix} + \varepsilon_t$$

where $\varepsilon_t \sim N(\emptyset, \Sigma)$ and $\Sigma = \begin{bmatrix} 0.00219 & 0.00162 \\ 0.00162 & 0.00167 \end{bmatrix}$. As in Bianchi (2011), we discretize the above process into a Markov process with 16 pairs of $\ln y^T$ and $\ln y^N$. The mean of each endowment is normalized to one. The endogenous state variable, d_t, is discretized with 100 evenly spaced points in the range from $0.4\,(1+r)$ and $1.02\,(1+r)$ which is

Table 1: Calibration

	Parameter	Value	Source
σ	Inverse of intertemporal elasticity of substitution	2	Bianchi (2011)
ξ	Intratemporal elasticity of substitution	0.83	
a	Weight of tradables	0.31	
β	Discount factor	0.91	
κ	Collateral	0.32	
r	Real interest rate	0.04	
χ	Weight of utility from public spending	0.02	Spending-to-GDP ratio

consistent with Bianchi (2011).

4.2 Borrowing Decisions

Figure 1 shows the borrowing choices as functions of the initial indebtedness. The decision rules are conditional on the worst state of exogenous endowment. Since the mean of the tradable endowment is normalized to one, decision rules can be interpreted as the ratios to the average level of the tradable endowment. We investigate decision rules under four alternative specifications of government spending policy: time-consistent optimal spending, the Samuelson rule (Samuelson, 1954), zero government spending policy, and constant government spending-to-GDP ratio policy. Under the Samuelson rule, the government spending is determined so that the marginal utility gain of higher public spending and marginal utility cost of lower private nontradable consumption are equated, meaning $u_N\left(c_t^T, y_t^N - g_t\right) = v'\left(g_t\right)$.[7] This policy attains the efficient provision of public goods and ignores the financial stability. We consider this policy to highlight the role of government spending in maintaining the financial stability. Note that the two terms that the Samuelson rule equates are the first two term of (25). By comparing the out-

[7]Under this policy, the size of government spending is a constant fraction of nontradable endowment in the case where tradable and nontradable consumption is separable in preferences.

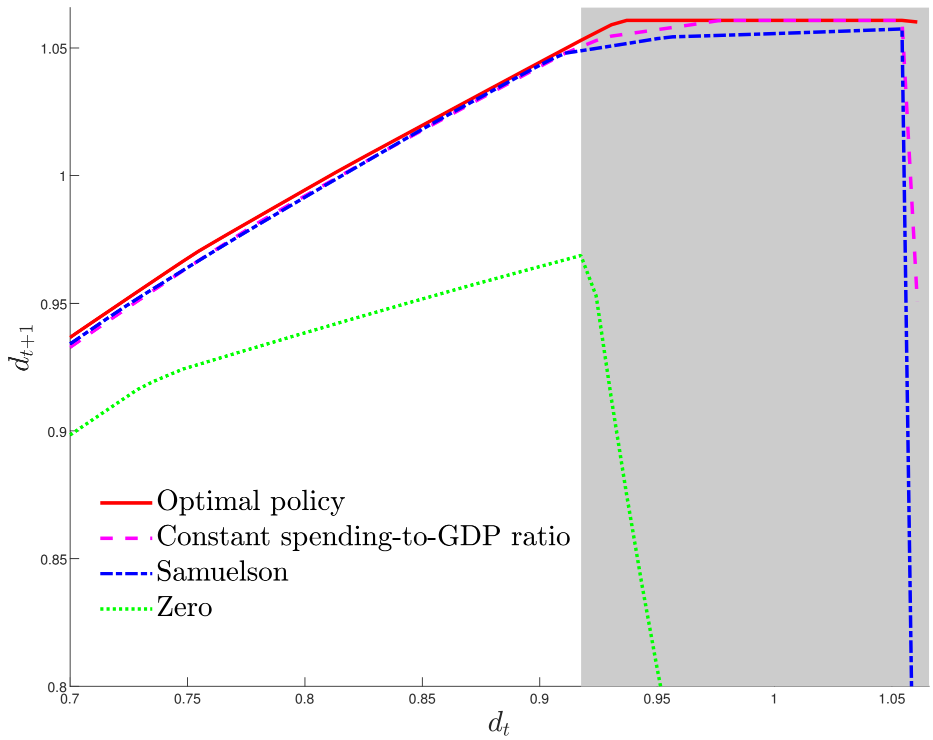
come under the optimal policy and under the Samuelson rule, we show the importance of mitigating the overborrowing problem and escaping from the debt-deflation spiral. Also, we consider the zero government spending policy. By comparing the outcome under the Samuelson rule and under the zero spending policy, we illustrate the benefit of sticking to the public provision of public goods. As argued, any positive level of government spending helps to maintain financial stability since it inflates the price of collateral and helps the economy with avoiding hitting the upper limit of borrowing. Thus, sticking to the efficient provision of public goods has benefits since as long as spending is conducted it contributes to financial stability. We consider the fourth specification of spending policy to propose the implementable second-best policy that mimics the optimal policy. The government spending-to-GDP ratio in current small open economy is given by $\frac{p_t g_t}{y_t^T + p_t y_t^N}$. Since the government should increase its spending to keep this ratio constant when the relative price falls, this policy realizes the fiscal stimulus when the collateral constraint holds with equality.[8]

With zero government spending, the decision rule of the borrowing shows the nonlinearity. In the region where the collateral constraint is slack, the level of borrowing is increasing in the borrowing in the previous period. However, in the region where the collateral constraint holds with equality (shaded region), the sign of the slope changes and it shows a significant downward-sloping pattern. The higher borrowing in the previous period induces a decrease in the tradable consumption given any level of the current level of borrowing. The smaller tradable consumption implies the lower relative price of nontradable, which leads to the lower value of the collateral. When the collateral constraint holds with equality, the lower value of collateral implies a lower level of borrowing.

Under the Samuelson rule, the economy enjoys a higher level of borrowing in all the regions than the economy with zero government spending and the collateral constraint binds in more narrow regions. It is shown that the decision rule shows the monotonically increasing pattern even in the highly indebted region where the collateral constraint binds under the zero spending policy. This is possible because a positive level of government

[8]We set the spending-to-GDP ratio at its average ratio under the optimal policy.

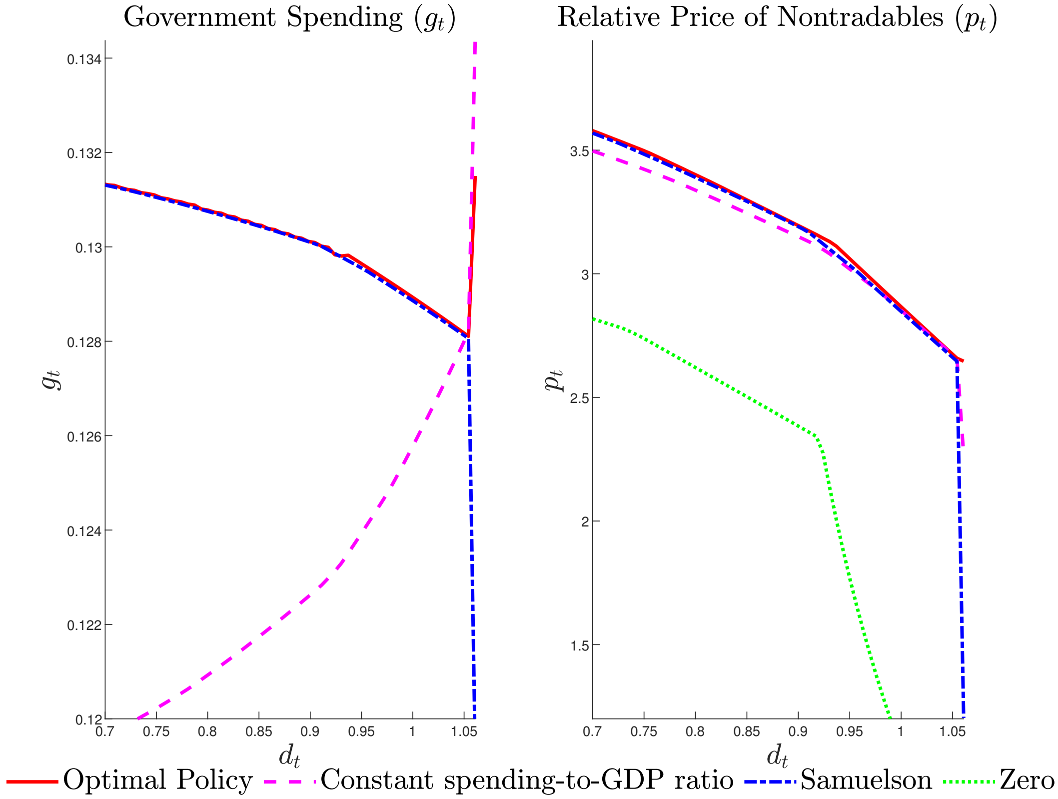
Figure 1: Borrowing Choices



Note: The figure shows the decision rules of borrowing under alternative government spending policies. The solid red line corresponds to the decision rule under the optimal policy, magenta to the constant spending-to-GDP ratio policy, blue to under the Samuelson rule, and green to under zero government spending. The collateral constraint binds under the zero spending policy in the shaded region.

spending, supports the price of collateral as a byproduct of the efficient provision of public goods. Under the same level of indebtedness, the value of collateral is higher with government spending than with zero government spending. Thus the spending allows higher borrowing without hitting its upper bound. The decision rule still shows the downward-sloping pattern, implying the binding of the collateral constraint, but in a very narrow region. Furthermore, the expectation of a lower probability of binding results in the higher borrowing ex ante, meaning that the level of borrowing is higher with the current specification of spending than with zero government spending in the region where the collateral constraint is slack with zero spending. The lower expectation for binding reduces the marginal value of saving of households and encourages borrowing for more current consumption.

Figure 2: Government Spending and Relative Price



Note: The figure presents the decision rules of the government spending under each policy specification and the corresponding relative price of nontradable as a function of the state of indebtedness. The decision rules are conditional on the worst state of endowments.

Among all specifications of spending, the optimal spending allows the highest level of borrowing in the whole region. Also, the significant downward-sloping pattern in the decision rule of borrowing is not observed with the optimal policy. Even in the highly indebted regions where the collateral constraint binds with other specifications of spending policy, the optimal spending policy allows for maintaining a high level of capital inflows. This property results from the incentive to maintain the financial stability which let the role of government spending go beyond the efficient provision of public goods as argued above. To see the point, Figure 2 presents the decision rules of optimal government spending under each specification of spending, and the corresponding relative price of nontradables as a function of current indebtedness. The decision rule for the optimal spending shows the kink and it is increasing in the current indebtedness in the highly indebted region where the collateral constraint binds absent the optimal policy. This fiscal stimulus is motivated by the utility benefit from increasing the borrowing capacity by appreciating the price of collateral. This policy decision prevents the relative price from falling sharply together with more indebtedness in the region where the collateral constraint holds with equality. Thus, the optimal fiscal stimulus when the level of borrowing hits its upper limit allows further borrowing. In contrast, under the Samuelson rule, the government spending is monotonically decreasing in the current indebtedness and the relative price exhibits a steep downward-sloping pattern in the region where the collateral constraint is binding. Note that government spending under the optimal policy does not deviate from the one under the Samuelson rule when the collateral constraint is slack. This implies that the gain from mitigating the overborrowing ex ante is not so large that the optimal policy sticks to the efficient provision of public goods. However, the optimal policy still allows the higher borrowing in all the region. The lower expectation for the binding of the collateral constraint driven by ex post fiscal stimulus leads to higher borrowing ex ante, meaning higher borrowing even in the region the collateral constraint is far from binding. Summing up, ex ante, the optimal policy reduces the possibility to hit the upper limit of borrowing with the level of spending consistent with the efficient provision of public goods, and ex post, it mitigates the debt-deflation spiral by appreciating the price of collateral with further stimulus.

The constant government spending-to-GDP ratio policy mimics the optimal fiscal stimulus when the borrowing hits its upper limit. The borrowing decision rule shows the downward-sloping pattern, but its slope is smaller compared with the one under the Samuelson rule, meaning that it allows higher borrowing for a given level of indebtedness in the region where the borrowing hits its upper limit. The higher level of borrowing is achieved via the fiscal stimulus and moderate deflation of the relative price as depicted in Figure 2.

The long-run simulation of artificial data reveals that the fiscal spending allows the higher level of borrowing without the binding of the collateral constraint. We simulate the model under each specification of government spending policy for one million years using the same sequence of exogenous endowments and the same initial value of borrowing.

Table 2: Long-run Average

	Optimal	Constant Spending-to-GDP Ratio	Samuelson	Zero
Average shadow value	0.005	0.012	0.016	0.017
Average level of borrowing	1.05	1.04	1.03	0.94

Note: The average shadow value is the long-run average of the shadow value of the collateral constraint.

Table 2 summarizes the average statistics. The long-run average of the shadow value of the collateral constraint is the lowest under the optimal policy among alternative specifications of spending policy with the highest average level of borrowing. Sticking to the efficient provision of public goods when the collateral constraint is slack allows the higher level of borrowing without hitting the upper limit, and the ex post fiscal stimulus makes it possible to borrow even when the level of borrowing is hitting the upper limit. Even without ex post stimulus, the Samuelson rule allows higher borrowing with lower distortion in intertemporal decisions compared with the case without any policy intervention.

4.3 Fiscal Stimulus during Financial Crises

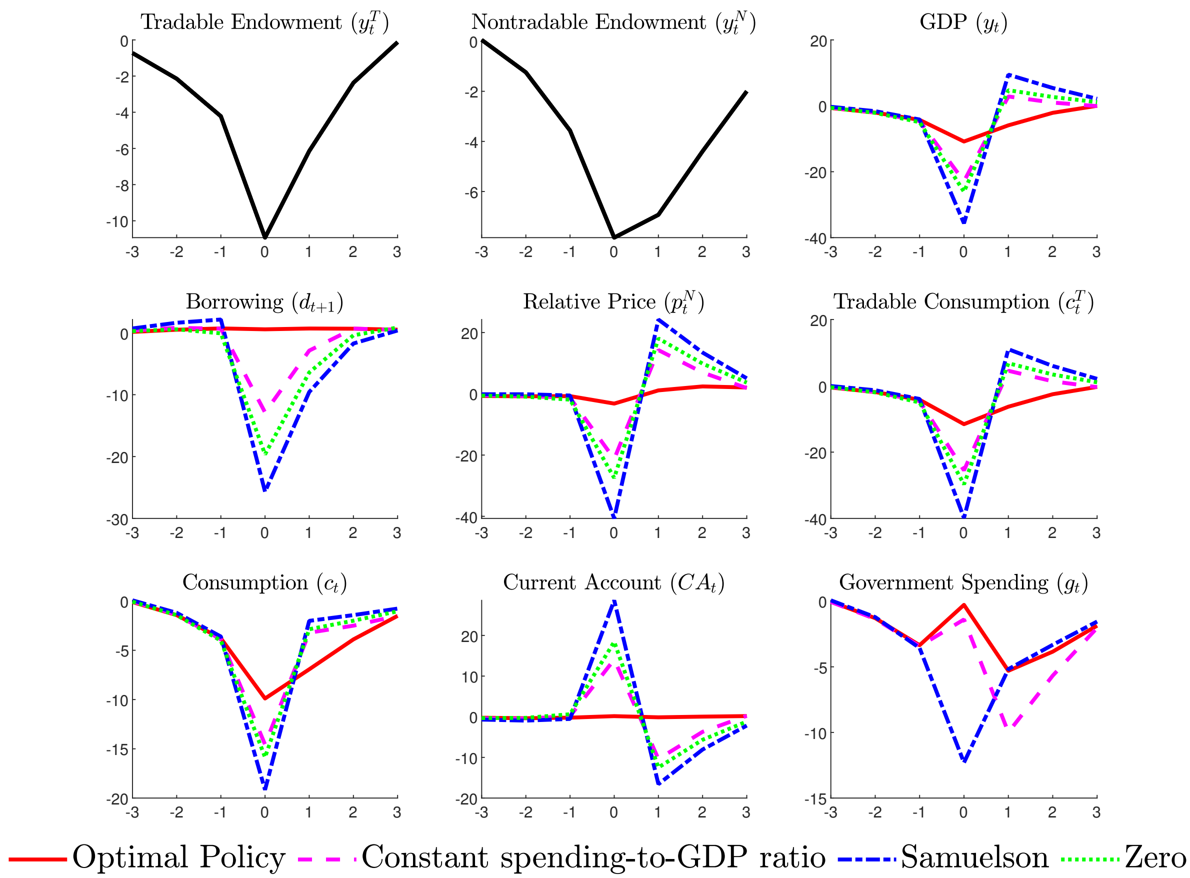
In this section, we highlight the role of government spending during financial crises. We simulate the model under each specification of government spending policy for one million years using the same sequence of exogenous endowments and the same initial value of borrowing. We define the period of a financial crisis as a period in which the current account is one standard deviation above its mean and the collateral constraint binds. The model generates 52593 periods of financial crisis under the zero spending policy, 44925 periods under the constant spending-to-GDP ratio policy, 31018 periods under the Samuelson rule, and 5 periods under the optimal policy. The positive level of government spending reduces the possibility of crises and the optimal spending almost completely avoids crises.

To examine the macroeconomic dynamics under each government spending policy, we identify the 7-year window of simulated exogenous endowments centered around a period in which crises happen under the Samuelson rule. We examine the macroeconomic dynamics under these specified realizations of endowments with each spending policy. Figure 3 presents the average dynamics of macroeconomic variables under each specification of government spending policy during the identified financial crises. The period in which a financial crisis happens is normalized to period zero. On average, under the Samuelson rule, a financial crisis happens when the tradable and nontradable output fall around 11 percent and 8 percent from their respective mean.

Without the optimal policy, capital inflow suddenly stops when the constraint binds, which results in the drastic reversal of the current account. The domestic absorption experiences severe reduction, which deflates the collateral price by 27 percent from its mean under the zero spending policy and sets the economy into a debt deflation spiral. Given the same realization of endowments, the crisis is more severe with the Samuelson rule than with zero government spending. Under the Samuelson rule, government spending falls more than the nontradable endowment during the crisis and thus the relative price of nontradable is deflated 41 percent from its mean which is more than the case where the government does not intervene. The procyclical spending under the Samuelson rule works to reduce the borrowing capacity of the economy during crises and amplifies recessions.

The optimal policy prevents the recessionary endowment shocks from resulting in financial crises in the sense that the drastic reversal of the current account is not observed. The optimal spending increases when the level of borrowing hits its upper limit following negative endowment shocks and decreases once the economy starts to recover with the increasing path of endowments. The fall in the relative price in period zero is only 3 percent on average. The fiscal stimulus props up the price of collateral and prevents the economy from falling into the debt-deflation spiral. The level of capital inflows is maintained, which avoids the catastrophic impacts of the recessionary shocks on other macroeconomic variables. Even though in our environment there is no nominal frictions, the optimality of countercyclical government spending is suggested. The constant spending-to-GDP ratio policy also conducts fiscal stimulus during financial crises, which results in the most modest crises among suboptimal policies.

Figure 3: Financial Crisis Dynamics



Note: Each panel shows the dynamics of each variable under financial crises. The red line represents dynamics under optimal government spending, magenta is those under the constant spending-to-GDP policy, blue is those under the Samuelson rule, and green is those under zero government spending. The dynamics are expressed in percentage deviation from respective long-run averages except for the current account which is expressed in absolute deviation from its long-run mean.

4.4 Welfare Analysis

We evaluate the welfare gain of optimal government spending policy against suboptimal policies. Our measure of welfare gain is the required percent increase in private and public consumption in economies with alternative policies to give the equivalent level of welfare as in the economy with optimal policy. The state-dependent welfare gain of optimal policy denoted by $\gamma\left(d, y^T, y^N\right)$ solves

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\left[c_t^a \left(1+\gamma\left(d, y^T, y^N\right)\right)\right]^{1-\sigma}}{1-\sigma} + \chi \frac{\left[g_t^a \left(1+\gamma\left(d, y^T, y^N\right)\right)\right]^{1-\sigma}}{1-\sigma} \right] = V^{OP}\left(d, y^T, y^N\right)$$

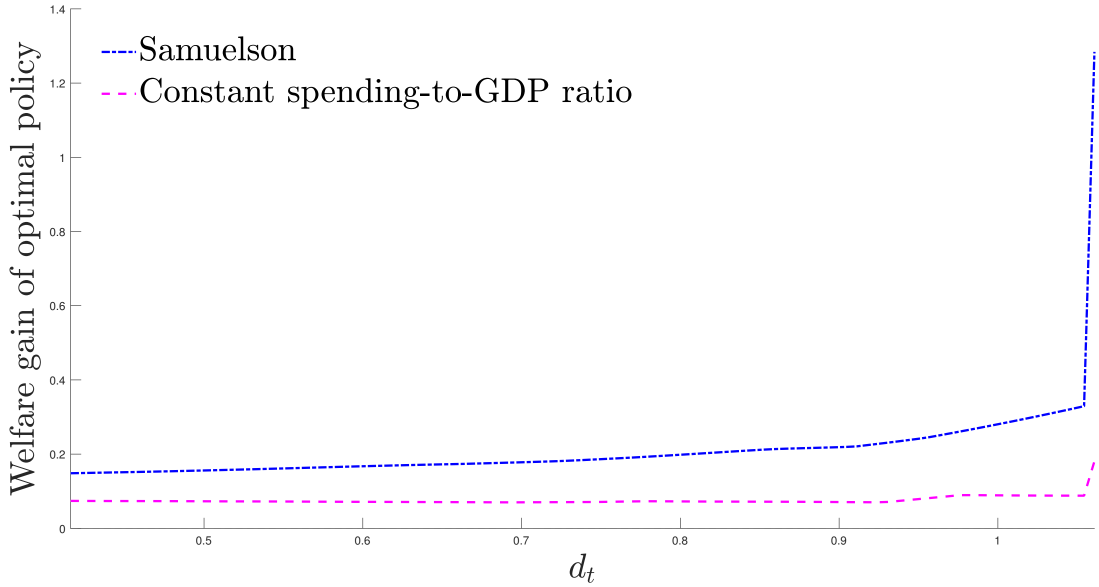
where c_t^a and g_t^a denote the private and public consumption under alternative spending policy and V^{OP} is the value function associated with the optimal government spending policy. Figure 4 presents the welfare gain of optimal policy as a function of initial indebtedness in the worst state of the exogenous endowments. The optimal policy delivers higher welfare in all the regions against the Samuelson rule and the constant spending-to-GDP ratio. The welfare gain is large in the region of high indebtedness in which the collateral constraint binds. This is because optimal fiscal stimulus when the collateral constraint holds with equality allows further borrowing and consumption. With the expectation of a lower possibility of the binding of collateral constraint under optimal policy, people have less incentive for precautionary saving. The less precautionary saving translates into the higher consumption and higher welfare in the region where the constraint is slack. Especially, since the Samuelson rule leads to the contraction of fiscal spending when the constraint binds that works to shrink the borrowing capacity of the economy, the welfare gain of optimal policy is larger against the Samuelson rule. The average welfare gain of optimal policy against the Samuelson rule, which is calculated with the distribution of the state under the Samuelson rule, is 0.23 percent.[9] Even though the optimal policy delivers the higher welfare at any indebtedness against the constant spending-to-GDP ratio policy, the gain is smaller since this suboptimal policy delivers fiscal stimulus when the level of borrowing hits its limit. The average gain which is 0.08 percent is also smaller.

5 Conclusion

This paper characterizes the optimal government spending in a small open economy with collateral constraints. The role of government spending extends beyond the efficient provision of public goods, especially when the collateral constraint is binding. By boosting demand in the nontradable sector and appreciating the price of collateral, optimal spending aims to mitigate the effects of the collateral constraint. When the constraint is slack, the spending allows for greater indebtedness without reaching the upper limit of borrow-

[9]This welfare gain is larger than the welfare gain of correcting the pecuniary externality, which is 0.135 percent reported by Bianchi (2011).

Figure 4: Welfare gain of optimal policy against suboptimal policies



Note: The figure presents the welfare gain of optimal policy against other specifications of government spending as a function of the current indebtedness conditional on the worst state of the exogenous endowment. The magenta and blue line correspond to the welfare gain against the constant spending-to-GDP ratio policy and the Samuelson rule.

ing. When the collateral constraint is binding, fiscal stimulus alleviates the debt-deflation spiral by appreciating the value of collateral. The optimal spending reduces the intertemporal distortion arising from the collateral constraint, preventing recessionary shocks from triggering financial crises and almost entirely avoiding financial crisis events in our simulation. A policy of maintaining a constant government spending-to-GDP ratio closely approximates the optimal government spending in our setting.

This paper highlights the stabilizing role of government spending in business cycles, advocating for the optimality of countercyclical fiscal spending during financial crises. This complements the Keynesian view of the optimality of countercyclical fiscal spending, which is often not observed in emerging economies (Kaminsky et al., 2005). For future research, it would be valuable to extend the model to explore the unexplored benefits of government spending in the context of sovereign debt, particularly for currency unions, as illustrated by Gourinchas et al. (2023).

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Appendix

A Optimal Time-Consistent Policy under Discretion

We denotes the multiplier on (20), (21), (22), (23) and (24) by λ^1, λ^2, λ^3, λ^4, and λ^5. The government's optimality conditions are the following:

$$\begin{aligned}
c_t^T :& u_T\left(c^T, y^N - g\right) - \lambda^1 + \left(\lambda^3 + \lambda^4\mu\right)\kappa\frac{\partial p}{\partial c^T}y^N + \frac{\lambda^2}{1+r}\frac{\partial u_T\left(c^T, y^N - g\right)}{\partial c^T} = 0, \\
d_{t+1} :& u_T\left(c^T, y^N - g\right) = \beta\left(1+r\right)E_{y^{T\prime}, y^{N\prime}|y^T, y^N}\Bigg[u_T\left(c^{T\prime}, y^{N\prime} - g'\right) + \lambda^{2\prime}\frac{\partial u_T\left(c^{T\prime}, y^{N\prime} - g'\right)}{\partial c^{T\prime}}\frac{1}{1+r} \\
&+ \left(\lambda^{3\prime} + \lambda^{4\prime}\mu'\right)\kappa\frac{\partial p'}{\partial c^{T\prime}}y^{N\prime} + \lambda^2\Omega' - \lambda^2\frac{\partial u_T\left(c^T, y^N - g\right)}{\partial c^T}\frac{1}{1+r} + \left(1 + r - \kappa\frac{\partial p}{\partial c^T}y^N\right)\left(\lambda^3 + \lambda^4\mu\right)\Bigg], \\
g_t :& \frac{\partial u\left(c^T, y^N - g\right)}{\partial g} + \chi g^{-\sigma} + \left(\lambda^3 + \lambda^4\mu\right)\kappa\frac{\partial p}{\partial g}y^N + \frac{\lambda^2}{1+r}\frac{\partial u_T\left(c^T, y^N - g\right)}{\partial g} = 0, \\
\mu_t :& -\lambda^2 + \lambda^4\left[\kappa\left(y^T + \frac{1-a}{a}\left(\frac{c^T}{y^N - g}\right)^{\frac{1}{\xi}}y^N\right) - d'\right] + \lambda^5 = 0, \qquad (26)
\end{aligned}$$

where $\Omega_{t+1} = \left[a\left(\boldsymbol{C^T}\left(d', y^{T\prime}, y^{N\prime}\right)\right)^{1-\frac{1}{\xi}} + (1-a)\left(y^{N\prime} - \boldsymbol{G}\left(d', y^{T\prime}, y^{N\prime}\right)\right)^{1-\frac{1}{\xi}}\right]^{\frac{\frac{1}{\xi}-\sigma}{1-\frac{1}{\xi}}} \frac{\partial a\left(\boldsymbol{C^T}\left(d', y^{T\prime}, y^{N\prime}\right)\right)^{-\frac{1}{\xi}}}{\partial d'} +$
$a\left(\boldsymbol{C^T}\left(d', y^{T\prime}, y^{N\prime}\right)\right)^{-\frac{1}{\xi}} \left[\frac{\partial\left[a\left(\boldsymbol{C^T}\left(d', y^{T\prime}, y^{N\prime}\right)\right)^{1-\frac{1}{\xi}} + (1-a)\left(y^{N\prime} - \boldsymbol{G}\left(d', y^{T\prime}, y^{N\prime}\right)\right)^{1-\frac{1}{\xi}}\right]^{\frac{\frac{1}{\xi}-\sigma}{1-\frac{1}{\xi}}}}{\partial d'}\right],$

$$\lambda^3 \left[\kappa\left(y^T + \frac{1-a}{a}\left(\frac{c^T}{y^N - g}\right)^{\frac{1}{\xi}} y^N\right) - d'\right] = 0,$$

and

$$\lambda^5 \mu = 0. \tag{27}$$

The multiplier on the private Euler equation is zero ($\lambda^2 = 0$) when the collateral constraint binds. Suppose $\mu > 0$. Then, $\kappa\left(y^T + \frac{1-a}{a}\left(\frac{c^T}{y^N-g}\right)^{\frac{1}{\xi}} y^N\right) - d' = 0$. The Kuhn-Tucker condition (27) implies $\lambda^5 = 0$. The optimality condition with respect to μ (26) imply $\lambda^2 = 0$.

B Numerical Solution Method

We solve for the time-consistent optimal policy with a nested fixed point algorithm as in Bianchi and Mendoza (2018) and Coulibaly (2023). The solution method consists of two loops. In the inner loop, the value function iteration gives us the value function and policy functions given future policies. Given these solutions, we update future policies in the outer loop.

1. We generate the 100 equally-spaced discrete grid for the borrowing. We employ linear interpolation to evaluate functions outside the grid. We initialize policy functions $c^T\left(d, y^T, y^N\right)$ and $g\left(d, y^T, y^N\right)$, and the value function $V\left(d, y^T, y^N\right)$ with those obtained for the competitive equilibrium with zero government spending. We use the same initial guess of future policies $C^T\left(d, y^T, y^N\right)$ and $G\left(d, y^T, y^N\right)$.

2. For each grid point we solve the government problem assuming the collateral constraint is not binding. With the assumption of $\mu = 0$, the problem is to solve

the Bellman equation (18) subject to (19), (20) and (21), given future policies $C^T\left(d, y^T, y^N\right)$ and $G\left(d, y^T, y^N\right)$. We check if (22) holds or not. If not, we solve the problem assuming the constraint is binding. The problem is to solve the Bellman equation (18) subject to (19), (20), (21), (24) and (22) holding with equality given future policies. Then we check the convergence of the value function. If the current and guessed value functions are not close enough, we update the value function.

3. We compare the solutions from inner loop, $c^T\left(d, y^T, y^N\right)$ and $g\left(d, y^T, y^N\right)$, and guessed future policies, $C^T\left(d, y^T, y^N\right)$ and $G\left(d, y^T, y^N\right)$. If they are not close enough, we update future policies and go back to the inner loop.

We solve for the competitive equilibrium under suboptimal policies with a time iteration algorithm as in Bianchi (2011).