Trade and the environment with a continuum of goods

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Abstract

A continuum of goods is introduced into the Ricardian model of trade between two countries, with each country endowed with an environmental stock that evolves over time and interacts with economic activities. The continuum of goods can be ranked in order of environmental sensitivity (how sensitive the productivity is to environmental changes), or environmental intensity (how harmful the production is to the environment). We examine the long-run environmental and welfare consequences of trade liberalization. We show that if two countries that share a similar ranking in environmental intensity open trade, one country suffers environmental degradation but another country gains from the improved environmental quality; otherwise, it is possible that in both countries the environmental quality either worsens or improves. We also show the welfare effects of trade can be decomposed into a green effect and a terms-of-trade effect, and that if trade openness leads to environmental degradation in both countries, both countries may lose from trade.

PRELIMINARY AND INCOMPLETE

1 Introduction

There has been much debate over the environmental consequences of trade. There are two contrasting views summarized in Copeland and Taylor (2003). One notion, called a pollution haven (PH) hypothesis, is that a shift of "dirty" industries occurs from countries in which strict pollution regulation is implemented to countries in which relatively lax pollution

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regulation is implemented. Since richer countries tend to have more stringent environmental standards and, by contrast, poorer countries have a comparative advantage in the dirty industries, trade reduces pollution in richer countries, while it increases pollution in poorer countries and total world pollution under the PH hypothesis. The other view is based on the factor endowment (FE) hypothesis, arguing that since polluting industries are also capital-intensive industries, countries that have higher capital endowments tend to have a comparative advantage in polluting industries and thus, export polluting goods. According to the FE hypothesis, richer countries, having tougher environmental standards than poorer countries and at the same time, having a lot of capital and thus, a comparative advantage in capital-intensive industries. Consequently, when trade is opened, capital-intensive industries expand in richer countries and contract in poorer countries, which means a shift of production of dirty industries from the richer to the poorer, and total world pollution decreases as a result.

The empirical evidence for the relationship between trade and pollution is also mixed. Antweiler et al. (2001) showed that overall in world trade data, high-income, capital-rich countries tend to be net exporters of polluting industries that produce a lot of emissions, and Dean and Lovely (2010) presented evidence, by using Chinese official environmental data on air and water pollution, and official trade data, that China's exports have shifted toward relatively cleaner sectors, and that the pollution intensity of Chinese exports has fallen dramatically between 1995 and 2004. These findings are in line with the FE hypothesis. On the other hand, using panel data covering the sulfur dioxide (SO2) and carbon dioxide (CO2) emissions of 88 countries from 1973 to 2000, Managi et al. (2009) showed that trade openness increases SO2 and CO2 emissions in non-OECD countries, while it decreases them in OECD countries, and using data on the annual rate of deforestation for 142 countries from 1990 to 2003, Tsurumi and Managi (2012) found that an increase in trade openness increases deforestation for non-OECD countries while slowing down deforestation for OECD countries. Levinson and Taylor (2008) also presented evidence for pollution haven effect by using data on U.S. regulations and trade with Canada and Mexico for 130 manufacturing industries from 1977 to 1986; they found that industries whose abatement costs increased most experienced the largest increases in net imports.

Both of the PH and FE hypotheses, however, assume pollution as disutility and no productivity effects of environmental quality on firms or industries. Governments may implement environmental policies such as pollution taxes or tradable pollution permits to enhance welfare, but the environmental protection costs are exogenous to firms since the tax rates or amounts of permits are determined by the governments. By contrast, if the environmental quality has a direct productivity effect, it can affect the firms' costs even in the absence of environmental policies. For example, agricultural production is highly sensitive to temperature, soil and water quality, and so on. Labor productivity in various sectors can also be affected by the environmental quality at the workplace. Moreover, in both of the PH and FE hypotheses, it is a common assumption that richer countries implement more stringent environmental policies than poorer countries. However, in reality, this may not be always the case, as we can see the United States under the Trump administration rolling back the climate policies under the Obama administration, while emerging economies such as China and India are introducing stringent environmental regulations; in particular, China's regulation on pollution emissions is now stricter than Japan's. Therefore, the difference in environmental policies across countries cannot necessarily be a determinant of trade patterns and environmental effects of trade.

In this paper, we develop a two-country, competitive trade model with environmental quality in each country as a stock variable affecting industries' productivity. We consider a continuum of industries that can differ in environmental sensitivity (i.e., how sensitive the productivity in an industry is to environmental changes) and in environmental intensity (i.e., how harmful the production in an industry is to the environment).[1] The environmental quality evolves over time, deteriorating as a result of pollution or resource extraction along with production, and improving as a result of natural depuration. In this setup of the model, we examine each country's comparative advantage and trade pattern, the long-run effects of trade liberalization on environmental quality and welfare in each country, and transition path of the environment from autarky to free trade. We show that if two countries that share a similar ranking in environmental intensity open trade, one country suffers environmental degradation but another country gains from the improved environmental quality; otherwise, it is possible that in both countries the environmental quality either worsens or improves. As for the welfare effects of trade, we decompose the total effect into a green effect caused by a change in the environmental quality and a terms-of-trade effect caused by a change in a country's terms of trade. The terms-or-trade effect is always positive, but the green effect can be negative, and we show that if trade openness leads to environmental degradation in both countries, it is possible that both countries lose from trade.

Applications of the Ricardian trade model with a continuum of goods, developed by Dornbusch et al. (1977) and further elaborated by Wilson (1980), to the analysis of trade and the environment are carried out by Copeland and Taylor (1994, 1995). The assumption of a continuum of goods is useful to analyze how trade affects pollution by changing a country's

[1] Note that environmental sensitive industries is not necessarily clean industries. As shown by Poore and Nemecek (2018), in which GHG emissions from beef production are on average several times higher as those from pork or chicken production, the environmental burden varies with industries.

composition of goods. Copeland and Taylor (1994, 1995) also assumed that governments set pollution policy endogenously and that environmental quality is a normal good, and these assumptions enable to investigate trade-induced technique effects. In the present study, we look at trading countries' composition of goods in a similar way, but do not consider endogenous environmental policies. Nevertheless, the environmental protection costs that firms face are endogenous even in the absence of environmental policies because we assume that environmental quality has a direct productivity effect. The assumption of the environmental stock affecting the productivity in an environmentally sensitive sector is also made in Copeland and Taylor (1999), Rus (2016), and Yanase and Li (2019). Copeland and Taylor (1999) developed a two-sector Ricardian model in which the production of "smokestack" manufacturing generates pollution, which lowers the productivity of an environmentally sensitive sector. Assuming a laissez-faire economy with pollution unregulated, they showed the extent to which trade may benefit both countries by spatially separating dirty and clean industries and thereby raising the world's production possibilities. By assuming that the environmentally sensitive sector also has a negative effect on the evolution of the environmental stock via e.g., resource extraction in a similar manner to the model of trade and renewable resources developed by Brander and Taylor (1997, 1998), Rus (2016) and Yanase and Li (2019) examined the effects of trade on the environment and welfare in a small-open economy and two-country world economy, respectively. Yanase and Li (2019), among others, show the possibility that both countries lose from trade, as in Karp et al. (2001) in a different framework.[2]

2 The model

There is a continuum of goods, indexed by $j \in [0, 1]$, and a single factor of production, labor. The markets are perfectly competitive, and the production technology is constant returns to scale with respect to labor:

$$Y(j) = \bar{A}(j)\, S^{\varepsilon(j)} L(j), \tag{1}$$

where $Y(j)$ is the output of good j, $L(j)$ is the amount of labor employed, and $\bar{A}(j)\, S^{\varepsilon(j)}$ measures the productivity of labor. The supply of labor is fixed exogenously at the level $\bar{L}$, and labor is freely mobile between industries within the country so that full employment holds:

$$\int_0^1 L(j)\, dj = \bar{L}.$$

[2]Karp et al. (2001) extended a North–South trade model developed by Chichilnisky (1994) in which differences in property rights for environmental resources create a motive for trade among otherwise identical countries.

The level of productivity is determined by two components: the exogenously given $\bar{A}(j)$ and the environmentally dependent $S^{\varepsilon(j)}$. The variable S denotes the environmental stock, and $\varepsilon(j) > 0$ indicates the environmental sensitivity of good j (by measuring how sensitive the productivity is to environmental changes). The goods are indexed in order of increasing environmental sensitivity:

$$\varepsilon'(j) > 0.$$

That is, good zero is least sensitive to environmental changes.

We assume that only from production processes arise the environmental impacts (e.g., pollution emissions or renewable resource extractions), the magnitude of which is in an industry-specific constant proportion to the output level:

$$E(j) = \lambda(j) Y(j), \tag{2}$$

where $E(j)$ is the environmental impacts arising from the production of good j, $\lambda(j)$ is the exogenously given impact-output ratio. The environmental stock evolves according to

$$\dot{S} = G(S) - E = G(S) - \int_0^1 E(j)\, dj, \tag{3}$$

where $G(S)$ is natural growth of the environment, and E is the aggregate environmental impacts. There exists a unique $K > 0$ such that $G(K) = 0$, where K can be interpreted as the carrying capacity of the environment (i.e., the capacity of the environment in the absence of economic activities).[3]

We also assume Cobb-Douglas demand:

$$U = \int_0^1 b(j) \ln C(j)\, dj \tag{4}$$

with $\int b(j)\, dj = 1$. The share of income spent on good j is $b(j)$ and the same at all prices.

Given the setup above, labor requirement for per unit output of good j is given by

$$a(j, S) = \frac{1}{\bar{A}(j) S^{\varepsilon(j)}}. \tag{5}$$

[3]Various specific forms of $G(S)$ have been used in the literature, such as the linear form $G(S) = g(K - S)$ in Copeland and Taylor (1999), the logistic form $G(S) = gS(K - S)$ in Brander and Taylor (1998), and the tent-shaped form in Benchekroun and Long (2016)

$$G(S) = \begin{cases} \delta S & S \leq \frac{g}{\delta+g} K \\ g(K - S) & S > \frac{g}{\delta+g} K \end{cases}.$$

It is convenient to define the environmental intensity by

$$e(j) \equiv \frac{E(j)}{L(j)} = \lambda(j)\,\bar{A}(j)\,S^{\varepsilon(j)},$$

which measures the environmental impacts caused by each unit of labor in the production of good j. If $e(j) > e(i)$, we say that good j is more environmentally harmful than good i.

3 Autarky

Serving as a comparison point to reveal the consequences of trade liberalization, this section considers a closed economy. We first analyze an economy without any government intervention, and then discuss an optimal policy that maximize the steady-state level of utility.

3.1 Laissez faire in autarky

In the short run, firms take the environmental stock as given. Profit maximization in perfectly competitive markets gives the price of goods by

$$p(j) = a(j,S)\,w,$$

where w is the wage level. The demand for good j is given by $C(j) = b(j)\,w\bar{L}/p(j) = b(j)\,\bar{L}/a(j,S)$, which together with the production function $Y(j) = \bar{A}(j)\,S^{\varepsilon(j)}L(j) = L(j)/a(j,S)$ gives

$$L(j) = b(j)\,\bar{L}. \tag{6}$$

The consumption level at each moment follows immediately:

$$C(j) = \bar{A}(j)\,S^{\varepsilon(j)}b(j)\,\bar{L} \tag{7}$$

The environmental stock evolves with time. At the steady state, $\dot{S} = 0$ holds and thus, using (3) and (6),

$$G(S) = \bar{L}\int_0^1 b(j)\,\lambda(j)\,\bar{A}(j)\,S^{\varepsilon(j)}dj, \tag{8}$$

from which we can solve for the steady-state level of environmental stock, henceforth denoted by S_{aut}. The stability of the steady state requires that

$$G'(S)\,S < \bar{L}\int_0^1 b(j)\,\varepsilon(j)\,\lambda(j)\,\bar{A}(j)\,S^{\varepsilon(j)}dj. \tag{9}$$

The utility level at autarky steady state can be expressed by, using (4) and 7,

$$U_{\text{aut}} = \underbrace{\ln \bar{L} + \int_0^1 b(j) \ln \left(\bar{A}(j)\, b(j) \right) dj}_{\text{exogenous}} + \left(\int_0^1 b(j)\, \varepsilon(j)\, dj \right) \ln S_{\text{aut}} \tag{10}$$

3.2 Autarkic optimal policy

Instead of jumping into the result directly, have a look at how an arbitrary level of environmental tax τ (imposed on each unit of environmental impacts) affects the economy.

In the competitive markets, the prices are equalized to the costs:

$$p(j) = \underbrace{a(j,S)\, w}_{\text{private cost}} + \underbrace{\tau \lambda(j)}_{\text{tax burden}}.$$

The total tax revenue is $T = \tau E$, which is transferred to households in a lump-sum fashion (so that households can afford all goods produced). Labor allocations is therefore

$$L(j) = \frac{b(j) \left(w\bar{L} + \tau E \right)}{p(j)\, \bar{A}(j)\, S^{\varepsilon(j)}} = \frac{w + \tau E/\bar{L}}{w + \tau e(j)} b(j)\, \bar{L},$$

and the consumption is

$$C(j) = a(j,S)\, \frac{w + \tau E/\bar{L}}{w + \tau e(j)} b(j)\, \bar{L}$$

The steady state is now characterized by

$$G(S) = \bar{L} \int_0^1 e(j)\, \frac{w + \tau E/\bar{L}}{w + \tau e(j)} b(j)\, dj,$$

from which we can solve for the steady-state level of environmental stock, denoted by $S_{\text{aut-tax}}$. The corresponding utility level at the steady state can be expressed by

$$U_{\text{aut-tax}} = \underbrace{\ln \bar{L} + \int_0^1 b(j) \ln \left(\bar{A}(j)\, b(j) \right) dj}_{\text{exogenous}} + \int_0^1 b(j) \ln \frac{w + \tau E/\bar{L}}{w + \tau e(j)} dj + \left(\int_0^1 b(j)\, \varepsilon(j)\, dj \right) \ln S_{\text{aut-tax}},$$

which clearly varies with the rate of environmental tax. The following result gives the optimal level of environmental tax:

Proposition 1. *In autarky, the steady-state level of utility can be maximized by imposing* *an environmental tax by letting*

$$\tau = \frac{\int_0^1 \varepsilon(j)\, b(j)\, dj}{\int_0^1 \frac{\lambda(j)}{p(j)} \varepsilon(j)\, b(j)\, dj - G'(S)\, S}$$

4 Trade liberalization

This section considers the trade between two countries, home and foreign (the rest of world), to both of which the above settings apply except for foreign functions and variables being indicated by superscript *. We let the wage rates w and w^* measured in common unit and, for simplicity, assume the identical preferences between home and foreign:

$$b(j) = b^*(j),$$

To highlight the interaction between trade and the environment, we also assume the identical technologies prevail in the two countries:

$$\bar{A}(j) = \bar{A}^*(j),$$
$$\varepsilon(j) = \varepsilon^*(j).$$

Since our focus in the paper is on the relatively local environmental problems such as soil contamination and air pollution, home and foreign's environments are described by two stock variables, S and S^*.

In what follows, we first analyze how the short-run trade equilibrium is determined (given environmental stocks), and consider the long-run equilibrium (steady state).

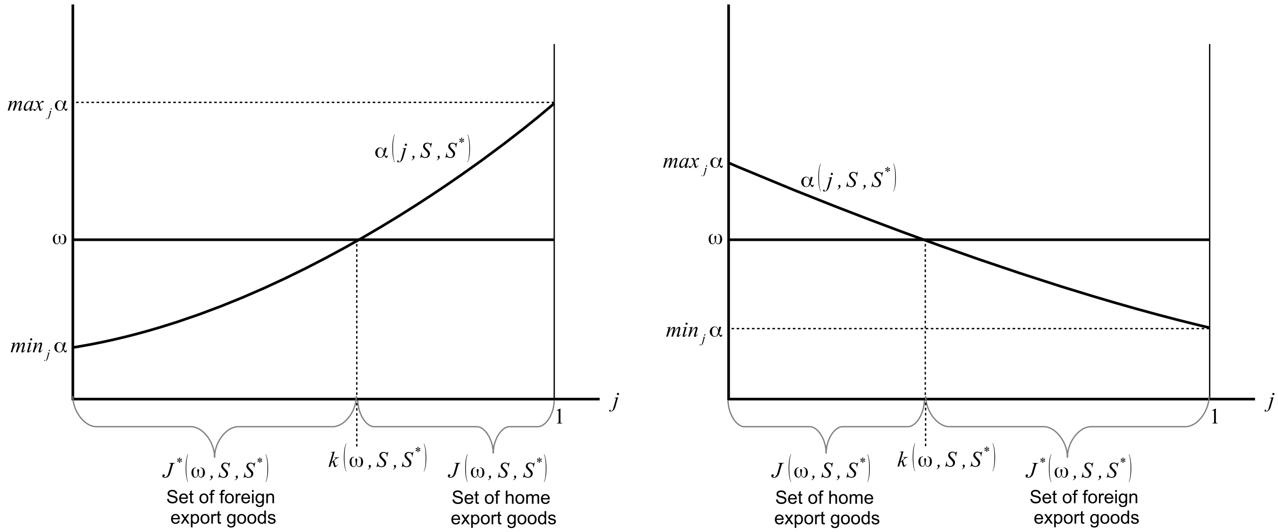
4.1 Trade equilibrium in the short run

To characterize the equilibrium under trade at every point in time, it requires to reveal trade patterns given the level of environmental stocks. For this purpose, it is convenient to define the relative wage rate (of home to foreign) by

$$\omega \equiv \frac{w}{w^*},$$

and the relative labor requirement (of foreign to home) by

$$\alpha(j, S, S^*) \equiv \frac{a^*(j, S^*)}{a(j, S)} = \left(\frac{S}{S^*}\right)^{\varepsilon(j)} \tag{11}$$



(a) $S > S^*$

(b) $S < S^*$

Figure 1: Trade pattern

Trade pattern can be revealed by comparing $\alpha(j,S,S^*)$ with ω: when $\alpha(j,S,S^*) > \omega\ (<\omega)$, good j will be exported from home to foreign (from foreign to home). This is because, say, if $\alpha(j,S,S^*) > \omega$, then $a(j,S)\,w < a^*(j,S^*)\,w^*$, which means that good j can be produced at a lower cost in home than is in foreign.

Having obtained the trade pattern, we can move on to the determination of trade equilibrium. For this purpose, it is convenient to define the set of home's export goods by

$$J(\omega,S,S^*) = \{j : \alpha(j,S,S^*) > \omega\}, \tag{12}$$

which is also the set of foreign's import goods. Similarly, the set of foreign's export goods (also the set of home's import goods) can be expressed by

$$J^*(\omega,S,S^*) = \{j : \alpha(j,S,S^*) < \omega\}. \tag{13}$$

As illustrated in Figure 1, the relative labor requirement $\alpha(j,S,S^*)$ is an increasing (decreasing) function of j for $S > S^*(< S^*)$, since we have indexed the goods such that $\varepsilon(j)$ is increasing with j. Given that $S \neq S^*$ and $\min_j \alpha \leq \omega \leq \max_j \alpha$, there exists a unique cut-off good k such that

$$\alpha(k,S,S^*) = \omega.$$

With this cut-off k, (12) and (13) can be expressed by, respectively,

$$J(\omega, S, S^*) = \begin{cases} (k, 1] & \text{if } S > S^* \\ [0, k) & \text{if } S < S^* \end{cases}, \tag{14}$$

and

$$J^*(\omega, S, S^*) = \begin{cases} [0, k) & \text{if } S > S^* \\ (k, 1] & \text{if } S < S^* \end{cases}. \tag{15}$$

Let θ denote the share of the world income spent on home produced goods, which can be derived from two aspects: comparative advantage and market clearing (or trade balance). Form the aspect of comparative advantage, any good $j \in J(\omega, S, S^*)$ is produced in home. This together with the C-D demand implies that

$$\theta = \int_{J(\omega,S,S^*)} b(j)\, dj, \tag{16}$$

which gives θ as a decreasing function of ω.[4] From the aspect of market clearing, the world market clearing condition, or equivalently, the balance of trade condition, requires that $w\bar{L} = \theta\left(w\bar{L} + w^*\bar{L}^*\right)$, from which we can obtain θ as an increasing function of ω:[5]

$$\theta = \frac{\omega\bar{L}}{\omega\bar{L} + \bar{L}^*}. \tag{17}$$

The trade equilibrium can be then derived by solving (16) and (17) for ω and θ, the (unique) solution of which is denoted by $\tilde{\omega}(S, S^*)$ and $\tilde{\theta}(S, S^*)$. Figure 2 draws the comparative advantage schedule (16) and the trade balance schedule (17), the intersection of which gives the (unique) short-run trade equilibrium.

Knowing the equilibrium level of ω and θ, labor allocations in trade equilibrium follow

[4] Note that $\theta = 0$ if $\omega \geq \max_j \alpha(j, S, S^*)$ and $\theta = 1$ if $\omega \leq \min_j \alpha(j, S, S^*)$. It is also clear that $\partial\theta/\partial S > 0$ and $\partial\theta/\partial S^* < 0$.

[5] Note that the market clearing condition is equivalent to trade balance condition $(1-\theta)\, w\bar{L} = \theta w^*\bar{L}^*$.

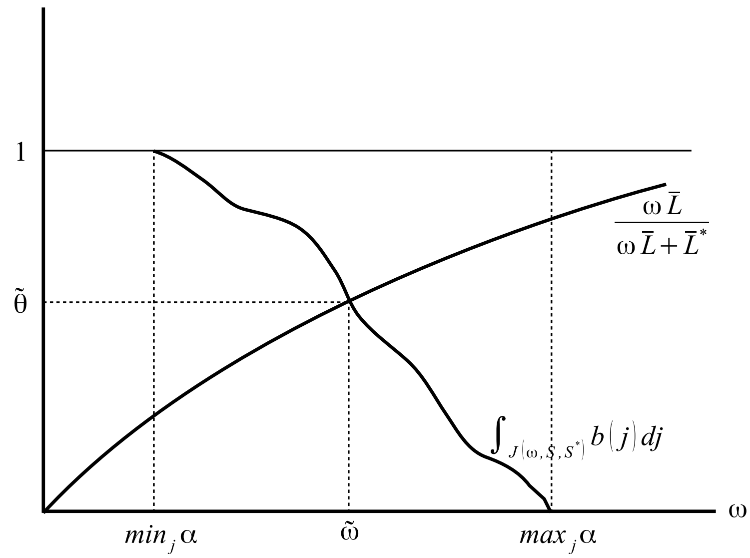


Figure 2: Trade equilibrium in the short run

immediately:

$$L(j) = \begin{cases} b(j)\left(\bar{L} + \frac{\bar{L}^*}{\tilde{\omega}}\right) = \frac{1}{\tilde{\theta}} b(j)\,\bar{L} & \text{if } j \in \tilde{J} \\ 0 & \text{if } j \in \tilde{J}^* \end{cases}, \tag{18}$$

$$L^*(j) = \begin{cases} 0 & \text{if } j \in \tilde{J} \\ b(j)\left(\tilde{\omega}\bar{L} + \bar{L}^*\right) = \frac{1}{1-\tilde{\theta}} b(j)\,\bar{L}^* & \text{if } j \in \tilde{J}^* \end{cases}, \tag{19}$$

where, and henceforth, $\tilde{\omega}$, $\tilde{\theta}$, $\tilde{J}$, $\tilde{J}^*$ are used for $\tilde{\omega}(S,S^*)$, $\tilde{\theta}(S,S^*)$, $J(\tilde{\omega}(S,S^*),S,S^*)$, $J^*(\tilde{\omega}(S,S^*),S,S^*)$ for simply notation. To see how to obtain (18) and (19), note that any good $j \in \tilde{J}$ will be produced in home, which means that $L^*(j) = 0$ and that

$$\underbrace{b(j)\left(w\bar{L} + w^*\bar{L}^*\right)}_{\text{world demand}} = \underbrace{p(j)\,Y(j)}_{\text{world supply}} = wL(j),$$

which together with $\tilde{\theta} = \tilde{\omega}\bar{L}/\left(\tilde{\omega}\bar{L} + \bar{L}^*\right)$ gives (18). On the other hand, any good $j \in \tilde{J}^*$ will be produced in foreign, which means $L(j) = 0$ and that

$$\underbrace{b(j)\left(w\bar{L} + w^*\bar{L}^*\right)}_{\text{world demand}} = \underbrace{p^*(j)\,Y^*(j)}_{\text{world supply}} = w^*L^*(j),$$

from which we can obtain (19).

Consumption in trade equilibrium can be obtained by noting that, given the identical preferences, a share $\tilde{\theta}$ of the goods, either produced in home or foreign, will be consumed by home households. That is,

$$C(j)=\begin{cases}\tilde{\theta}Y(j)=\bar{A}(j)\,S^{\varepsilon(j)}b(j)\,\bar{L} & \text{if } j\in\tilde{J}\\ \tilde{\theta}Y^{*}(j)=\frac{\tilde{\theta}}{1-\tilde{\theta}}\bar{A}(j)\,S^{*\varepsilon(j)}b(j)\,\bar{L}^{*} & \text{if } j\in\tilde{J}^{*}\end{cases}. \tag{20}$$

Similarly, a share $1-\tilde{\theta}$ of the goods will be consumed by foreign households, which gives

$$C^{*}(j)=\begin{cases}\left(1-\tilde{\theta}\right)Y(j)=\frac{1-\tilde{\theta}}{\tilde{\theta}}\bar{A}(j)\,S^{\varepsilon(j)}b(j)\,\bar{L} & \text{if } j\in\tilde{J}\\ \left(1-\tilde{\theta}\right)Y^{*}(j)=\bar{A}(j)\,S^{*\varepsilon(j)}b(j)\,\bar{L}^{*} & \text{if } j\in\tilde{J}^{*}\end{cases}. \tag{21}$$

A distinguished feature is that the consumption levels of domestically produced goods are dependent only on domestic environment and the same as autarky (as long as the environmental stock remains the same).

Table 1 summarizes the comparative statics in the short-run trade equilibrium. Figure 2 is useful in deriving these results. For instance, when S increases or S^{*} decreases, the comparative advantage schedule in the figure shifts right, resulting in an increase in both $\tilde{\omega}$ and $\tilde{\theta}$.

	$\tilde{\omega}$	$\tilde{\theta}$	$C(j)$ for $j\in\tilde{J}$	$C(j)$ for $j\in\tilde{J}^{*}$	$C^{*}(j)$ for $j\in\tilde{J}$	$C^{*}(j)$ for $j\in\tilde{J}^{*}$
S	+	+	+	+	indeterminate	0
S^{*}	−	−	0	indeterminate	+	+

Table 1: Comparative statics in the short run

4.2 Long-run trade equilibrium (trade steady state)

As the environmental stocks changes with time, the short-run trade equilibrium is not necessarily sustainable in the long run. In this section, we consider the trade dynamics and the trade steady state.

The dynamic equations of the environmental stocks can be expressed by, using (3), the foreign correspondence, (18), and (19),

$$\dot{S}=G(S)-E=G(S)-\Omega_{J}(S,S^{*})\,\bar{L},$$
$$\dot{S}^{*}=G^{*}(S^{*})-E^{*}=G^{*}(S^{*})-\Omega^{*}_{J^{*}}(S,S^{*})\,\bar{L}^{*},$$

where

$$\Omega_J(S, S^*) \equiv \frac{1}{\tilde{\theta}} \int_{\tilde{J}} b(j)\, e(j)\, dj$$
$$\Omega^*_{J^*}(S, S^*) \equiv \frac{1}{1-\tilde{\theta}} \int_{\tilde{J}^*} b(j)\, e^*(j)\, dj$$

measure respectively home and foreign's weighted average environmental intensity of each's export goods.[6] At the trade steady state, we have

$$G(S) = \Omega_J(S, S^*)\, \bar{L},$$
$$G^*(S^*) = \Omega^*_{J^*}(S, S^*)\, \bar{L}^*,$$

from which we can solve for the steady-state level of environmental stocks, denoted by S_{tra} and S^*_{tra}.

To see how the environmental stocks change after trade liberalization, it is convenient to define

$$\Omega(S) \equiv \int_0^1 b(j)\, e(j),$$
$$\Omega^*(S^*) \equiv \int_0^1 b(j)\, e^*(j)\, dj,$$

which measures respectively home and foreign's average environmental intensity in autarky. The differences in the average environmental intensity between trade and autarky are then

$$\Omega_J(S, S^*) - \Omega(S) = \left(1-\tilde{\theta}\right)\left(\Omega_J(S, S^*) - \Omega_{J^*}(S, S^*)\right), \tag{22}$$
$$\Omega^*_{J^*}(S, S^*) - \Omega^*(S^*) = \tilde{\theta}\left(\Omega^*_{J^*}(S, S^*) - \Omega^*_J(S, S^*)\right). \tag{23}$$

where

$$\Omega_{J^*}(S, S^*) \equiv \frac{1}{1-\tilde{\theta}} \int_{\tilde{J}^*} b(j)\, e(j)\, dj,$$
$$\Omega^*_J(S, S^*) \equiv \frac{1}{\tilde{\theta}} \int_{\tilde{J}} b(j)\, e^*(j)\, dj,$$

can be interpreted respectively as home and foreign's *imputed* weighted average environmental intensity of each's *import* goods (calculated as if they were produced domestically). It then follows that:

[6] The name of "weighted average" comes from the fact that $\int_J b(j)\, dj/\tilde{\theta} = 1$ and $\int_{J^*} b(j)\, dj/\left(1-\tilde{\theta}\right) = 1$.

Proposition 2. *If a country's weighted average environmental intensity of export goods is greater (lower) than that of imports, trade liberalization harms (enhances) the country's environment.*

Proof. Given that the weighted average environmental intensity of export goods is greater (lower) than that of imports, according to (22) and (23), the environment under trade cannot sustain a level equal to or above (below) the autarky steady-state level. □

We can apply this proposition in the following relatively special situations and obtain

Corollary 1. *If the environmental intensities in both countries are monotonic, the environmental consequences of trade are as follows.*

1. *If $e'(j), e^{*\prime}(j) > 0$ or $e'(j), e^{*\prime}(j) < 0$, two countries' environmental stocks change in the opposite directions, i.e., increases in one country and decreases in the other after trade liberalization.*

2. *If $e^{*\prime}(j) < 0 < e'(j)$ or $e'(j) < 0 < e^{*\prime}(j)$, two countries' environmental stocks change in the same direction, i.e., increase or decrease in both countries after trade liberalization.*

Proof. See Appendix. □

The rest of this section focuses on the understanding the welfare effects of trade. For this purpose, we derive the following result:

Lemma 1. *At the trade steady state, home's utility level can be expressed by*

$$U_{tra} = \underbrace{\ln \bar{L} + \int_0^1 b(j) \ln \left(\bar{A}(j) b(j)\right) dj}_{exogenous} + \left(\int_0^1 b(j) \varepsilon(j) dj\right) \ln S_{tra} + \int_{\tilde{J}^*} b(j) \ln \left(\frac{\tilde{\omega}}{\alpha(j, S_{tra}, S^*_{tra})}\right) dj, \tag{24}$$

foreign's utility level can be expressed by

$$U^*_{tra} = \underbrace{\ln \bar{L}^* + \int_0^1 b(j) \ln \left(\bar{A}(j) b(j)\right) dj}_{exogenous} + \left(\int_0^1 b(j) \varepsilon(j) dj\right) \ln S^*_{tra} + \int_{\tilde{J}} b(j) \ln \left(\frac{\alpha(j, S_{tra}, S^*_{tra})}{\tilde{\omega}}\right) dj. \tag{25}$$

Proof. See Appendix. □

It then follows immediately that

Proposition 3. *The differences in each country's utility level between trade and autarky steady state are*

$$U_{tra} - U_{aut} = \underbrace{\left(\int_0^1 b(j)\,\varepsilon(j)\,dj\right)\ln\frac{S_{tra}}{S_{aut}}}_{\text{green effect}} + \underbrace{\int_{\tilde{J}^*} b(j)\ln\left(\frac{\tilde{\omega}}{\alpha(j, S_{tra}, S^*_{tra})}\right)dj}_{\text{terms-of-trade effect}>0}, \tag{26}$$

$$U^*_{tra} - U^*_{aut} = \underbrace{\left(\int_0^1 b(j)\,\varepsilon(j)\,dj\right)\ln\frac{S^*_{tra}}{S^*_{aut}}}_{\text{green effect}} + \underbrace{\int_{\tilde{J}} b(j)\ln\left(\frac{\alpha(j, S_{tra}, S^*_{tra})}{\tilde{\omega}}\right)dj}_{\text{terms-of-trade effect}>0}. \tag{27}$$

Proof. The result follows immediately from Lemma 1 and (10). □

As shown in the proposition, the welfare effects of trade can be decomposed into two components: the green effect and the terms-of-trade effect. The former is positive (negative) if the environment improves (degrades) after trade liberalization. The latter is, however, necessarily positive since otherwise the goods will not be imported. The name of "terms-of-trade" comes from the fact that, in (26),

$$\frac{\tilde{\omega}}{\alpha(j, S_{\text{tra}}, S^*_{\text{tra}})} = \frac{p(j)}{p^*(j)},$$

where $p(j) = a(j, S)\,w$ $(j \in J^*)$ can be interpreted as the imputed price of good j calculated as if produced domestically, and $p^*(j) = a^*(j, S^*)\,w^*$ $(j \in J^*)$ is the actual import price of the good. Noting that $p(j) > p^*(j)$ for $j \in J^*$, the second term in (26) measures the benefit of trade for home by allowing home to buy goods at cheaper prices from foreign than producing by itself. Similarly, in (27)

$$\frac{\alpha(j, S_{\text{tra}}, S^*_{\text{tra}})}{\tilde{\omega}} = \frac{p^*(j)}{p(j)}$$

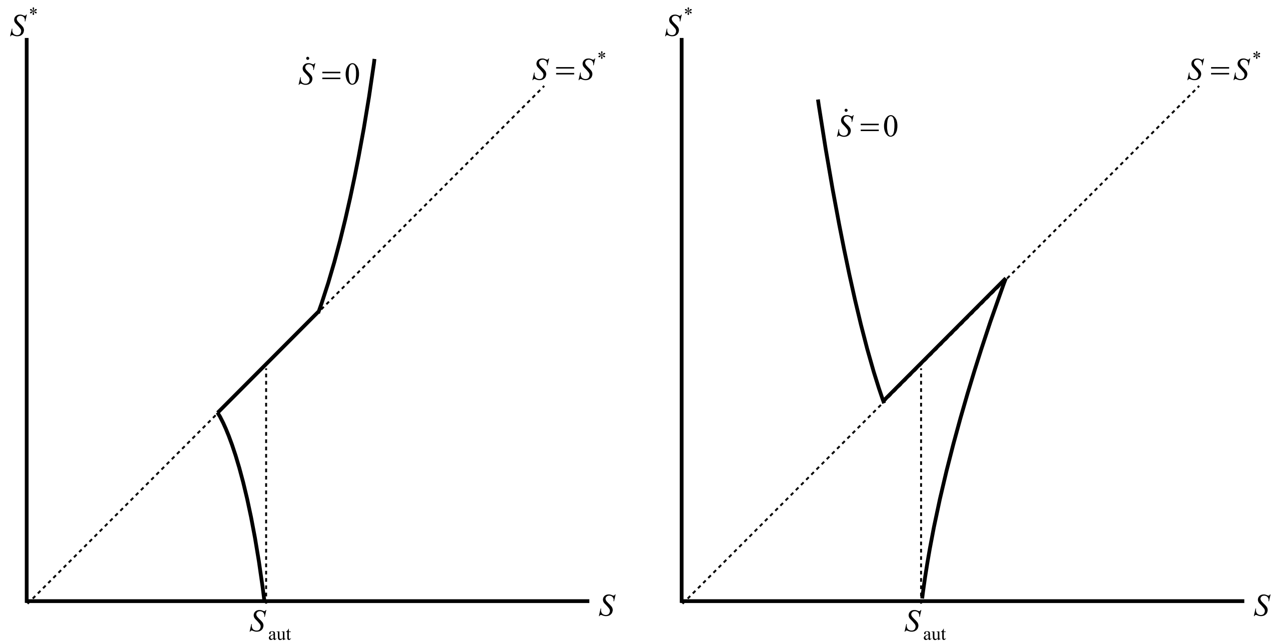
where $p^*(j) = a^*(j, S^*)\,w^*$ $(j \in J)$ can be interpreted as the imputed price of good j calculated as if produced in foreign, and $p(j) = a(j, S)\,w$ $(j \in J)$ is the actual import price of the good from home. Since $p^*(j) > p(j)$ for $j \in J$, the second term in (27) measures the benefit of trade for foreign by accessing cheaper goods in home.

Proposition 3 implies the following corollary:

Corollary 2. *The welfare effects of trade are as follows:*

1. *If the environment in both countries improves, both gain from trade.*

2. *If the environment improves in one country and degrades in the other, the former gain from trade and the latter may gain or lose.*



(a) $e'(j) > 0$ (b) $e'(j) < 0$

Figure 3: Home's $\dot{S} = 0$ curve

3. If the environment in both countries degrades, it is possible for both to lose from trade.

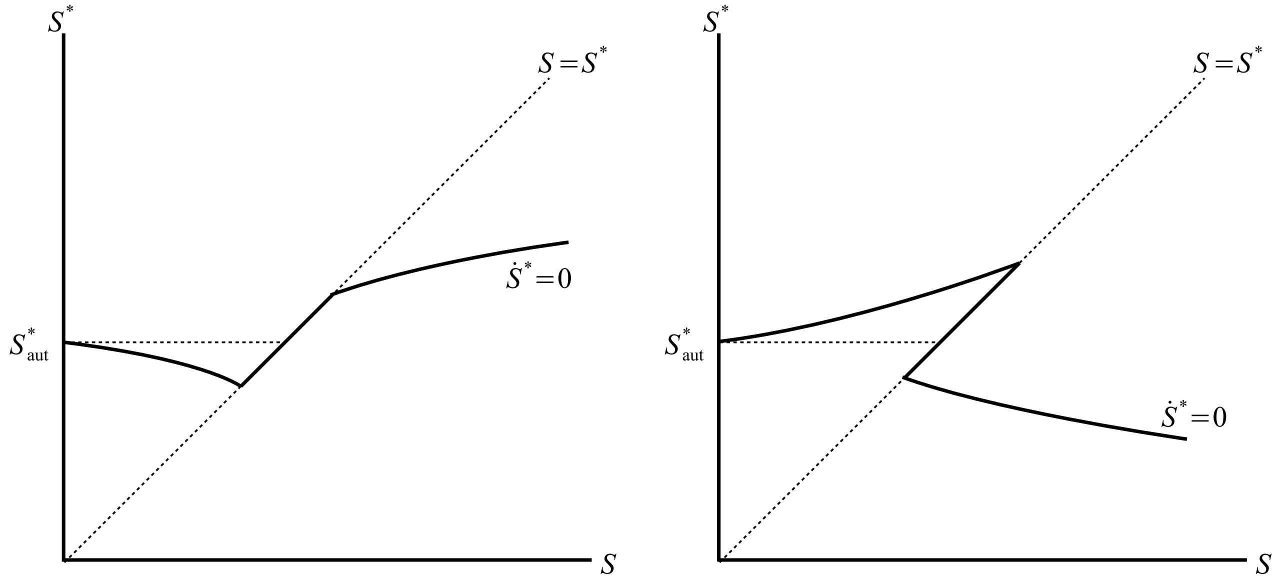
Proof. The results follow directly from (26) and (27). □

4.3 Phase diagram and transition dynamics

A phase diagram that draws the $\dot{S} = 0$ and $\dot{S}^* = 0$ curves on the (S, S^*) plane provides an intuitive exposition of how the trade steady state is determined and the transition dynamics to the steady state.

Here, for simple illustration, we focus on monotonic environmental intensities. Figure 3 gives home's $\dot{S} = 0$ curve for $e'(j) > 0$ and $e'(j) < 0$; Figure 4 depicts foreign's $\dot{S}^* = 0$ curve for $e^{*\prime}(j) > 0$ and $e^{*\prime}(j) < 0$. The detailed derivation of these curves is provided in Appendix.

Having derived the $\dot{S} = 0$ and $\dot{S}^* = 0$ curves, we can put them together to obtain the steady state and the transition dynamics. Clearly, even being confined in the relatively simple cases of monotonic environmental intensities, there are various possibilities including multiple steady states (which may arise when a country has a decreasing environmental intensity. Figure 5 provides an example, where $e'(j) > 0$ and $e^{*\prime}(j) > 0$, and compared to autarky, home's environment improves and foreign's degrades at the trade steady state.



(a) $e^{*\prime}(j) > 0$ (b) $e^{*\prime}(j) < 0$

Figure 4: Foreign's $\dot{S}^* = 0$ curve

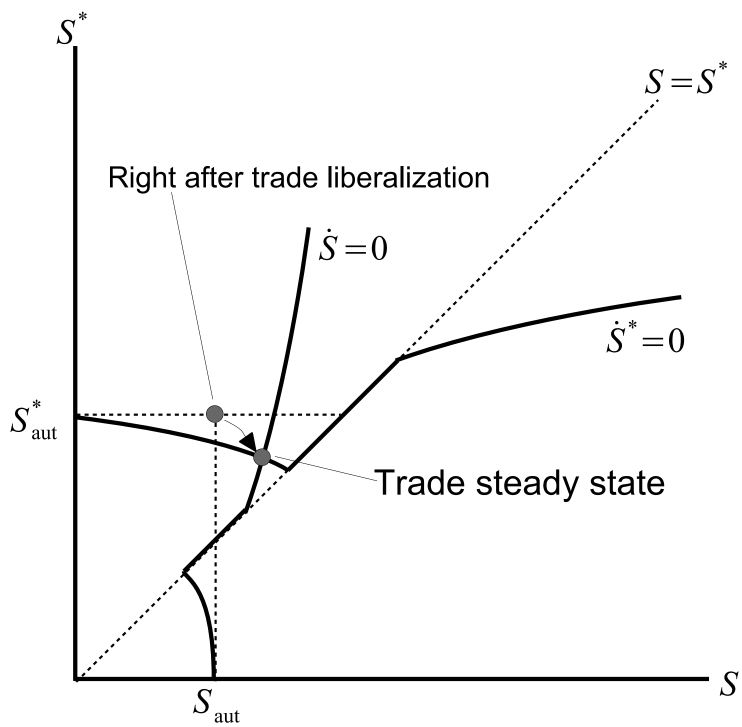


Figure 5: An example of phase diagram

5 Conclusion

This study developed a Ricardian model of trade between two countries, with a continuum of goods characterized by two aspects regarding the environment: how sensitive the productivity is to environmental changes (environmental sensitivity), and how harmful the production is to the environment (environmental intensity). Equipped with these features, our model provides a theoretical framework on the nexus between trade and the environment while environmental sensitivity and environmental intensity. We showed that the environmental consequences and welfare implications of trade are crucially dependent on the combination of the ranking in the two aspects between trading countries.

These findings highlight the significance of environmental regulation, as well as trade policies, regarding environmental preservation and trade benefits. An explicit consideration of the effects of policy interventions and the possible optimal combinations of these policies are left for future research.

6 Appendix

6.1 Proof of Proposition 1

Social planner problem Consider the following social planner problem:

$$\max \int_0^1 b(j) \ln C(j)\, dj,$$

subject to

$$\begin{aligned} C(j) &= \bar{A}(j)\, S^{\epsilon(j)} L(j)\,, \\ \bar{L} &= \int_0^1 L(j)\, dj, \\ G(S) &= \int_0^1 \lambda(j)\, \bar{A}(j)\, S^{\varepsilon(j)} L(j)\, dj. \end{aligned}$$

The Lagrangian can be written as

$$\begin{aligned} \mathcal{L} = \int_0^1 b(j) \ln C(j)\, dj + \int_0^1 p(j) \left(\bar{A}(j)\, S^{\varepsilon(j)} L(j) - C(j)\right) dj \\ + \mu \left(\bar{L} - \int_0^1 L(j)\, dj\right) + \gamma \left(G(S) - \int_0^1 \lambda(j)\, \bar{A}(j)\, S^{\varepsilon(j)} L(j)\, dj\right). \end{aligned}$$

The first order necessary conditions are

$$\frac{\partial \mathcal{L}}{\partial C(j)} = 0 \text{ for all } j,$$
$$\frac{\partial \mathcal{L}}{\partial L(j)} = 0 \text{ for all } j,$$
$$\frac{\partial \mathcal{L}}{\partial S} = 0,$$

from which we can obtain

$$p(j) = \frac{b(j)}{C(j)} \text{ for all } j, \tag{28}$$
$$\mu = (p(j) - \gamma\lambda(j))\, \bar{A}(j)\, S^{\varepsilon(j)} \text{ for all } j, \tag{29}$$
$$\gamma = \frac{\int_0^1 \varepsilon(j)\, b(j)\, dj}{\int_0^1 \frac{\lambda(j)}{p(j)} \varepsilon(j)\, b(j)\, dj - G'(S)\, S}. \tag{30}$$

These conditions and constraints above characterize $L(j)$, S, μ, and γ in the social optimal. Specifically, the shadow prices of goods can be expressed by, using (29),

$$p(j) = \frac{\mu}{\bar{A}(j)\, S^{\varepsilon(j)}} + \gamma\lambda(j). \tag{31}$$

It follows from (28) that $\bar{A}(j)\, S^{\varepsilon(j)} L(j) = b(j)/p(j)$, which yields labor allocation:

$$L(j) = \frac{b(j)}{p(j)\, \bar{A}(j)\, S^{\varepsilon(j)}} = \frac{b(j)}{\mu + \gamma\lambda(j)\, \bar{A}(j)\, S^{\varepsilon(j)}} = \frac{b(j)}{\mu + \gamma e(j)}. \tag{32}$$

It also follows from (32) that $\int_0^1 b(j)\, dj = \int_0^1 (\mu + \gamma e(j))\, L(j)\, dj$, which gives the following aggregate condition:

$$1 = \mu\bar{L} + \gamma E. \tag{33}$$

Combining (32) and (33) yields that

$$L(j) = \frac{1}{\mu\bar{L} + \gamma e(j)\, \bar{L}} b(j)\, \bar{L} = \frac{1}{\mu\bar{L} + \gamma E - \gamma E + \gamma e(j)\, \bar{L}} b(j)\, \bar{L} = \frac{1}{1 + \gamma\bar{L}\left(e(j) - \frac{E}{\bar{L}}\right)} b(j)\, \bar{L}. \tag{34}$$

Replicating the optimal in market To replicate the social optimal with market-based policies, note that the relative price of two goods in the economy ruled by the social planner satisfies

$$\frac{p_i}{p_j} = \frac{\frac{\mu}{\bar{A}(i)S^{\varepsilon(i)}} + \gamma\lambda(i)}{\frac{\mu}{\bar{A}(j)S^{\varepsilon(j)}} + \gamma\lambda(j)} = \frac{a(i,S)(\mu + \gamma e(i))}{a(j,S)(\mu + \gamma e(j))}, \tag{35}$$

with

$$\mu = (p(j) - \gamma\lambda(j))\,\bar{A}(j)\,S^{\varepsilon(j)} = \frac{p(j)}{a(j,S)} - \gamma e(j). \tag{36}$$

Now we can consider an ad-valorem tax τ on the environmental impacts (pollution or resource extraction). In the competitive market, the price of good j is equalized to its cost:

$$p(j) = \underbrace{a(j,S)\,w}_{\text{private cost}} + \underbrace{\tau\lambda(j)}_{\text{tax}} = \frac{w}{\bar{A}(j)\,S^{\varepsilon(j)}} + \tau\lambda(j),$$

which implies the social optimal can be achieved, i.e., (35) and (36) hold (with $w = \mu$), by letting

$$\tau = \gamma.$$

6.2 Proof of Corollary 1

Consider the case in which $S > S^*$ holds after trade openness. The case of $S < S^*$ can be similarly discussed (and thus omitted here).

If $e'(j) > 0$ and $e^{*\prime}(j) > 0$, then

$$e(i) > e(j) \text{ and } e^*(i) < e^*(j) \text{ for any } i \in J,\, j \in J^*,$$

which implies that

$$\Omega_J(S,S^*) > \Omega_{J^*}(S,S^*) \text{ and } \Omega^*_{J^*}(S,S^*) < \Omega^*_J(S,S^*).$$

That is, higher environmental burdens in home and lower burdens in foreign after trade openness.

If $e'(j) < 0$ and $e^{*\prime}(j) < 0$, then

$$e(i) < e(j) \text{ and } e^*(i) > e^*(j) \text{ for any } i \in J,\, j \in J^*,$$

which implies the opposite:

$$\Omega_J(S,S^*) < \Omega_{J^*}(S,S^*) \text{ and } \Omega^*_{J^*}(S,S^*) > \Omega^*_J(S,S^*).$$

That is, lower environmental burdens in home and higher burdens in foreign after trade openness.

In either case, the environment in one country degrades and that in the other improves.

If $e'(j) > 0$ and $e^{*\prime}(j) < 0$, then

$$e(i) > e(j) \text{ and } e^*(i) > e^*(j) \text{ for any } i \in J,\ j \in J^*,$$

which implies that

$$\Omega_J(S,S^*) > \Omega_{J^*}(S,S^*) \text{ and } \Omega^*_{J^*}(S,S^*) > \Omega^*_J(S,S^*).$$

That is, the environment in both countries improves.

If $e'(j) < 0$ and $e^{*\prime}(j) > 0$, then

$$e(i) < e(j) \text{ and } e^*(i) < e^*(j) \text{ for any } i \in J,\ j \in J^*,$$

which implies the opposite:

$$\Omega_J(S,S^*) < \Omega_{J^*}(S,S^*) \text{ and } \Omega^*_{J^*}(S,S^*) < \Omega^*_J(S,S^*).$$

That is, the environment in both countries degrades.

6.3 Proof of Lemma 1

It follows from (20) that home's utility level at the trade steady state satisfies

$$\begin{aligned}
U_{\text{tra}} &= \underbrace{\int_J b(j)\ln\left(\bar{A}(j)\, S_{\text{tra}}^{\varepsilon(j)} b(j)\, \bar{L}\right) dj}_{\text{utility from domesticly produced goods}} + \underbrace{\int_{J^*} b(j)\ln\left(\frac{\tilde{\theta}}{1-\tilde{\theta}}\bar{A}(j)\, S_{\text{tra}}^{*\varepsilon(j)} b(j)\, \bar{L}^*\right) dj}_{\text{utility from import goods}} \\
&= \int_0^1 b(j)\ln\left(\bar{A}(j)\, b(j)\right) dj + \left(\int_J b(j)\, dj\right)\ln \bar{L} + \left(\int_{J^*} b(j)\, dj\right)\ln\left(\frac{\tilde{\theta}}{1-\tilde{\theta}}\bar{L}^*\right) \\
&\qquad + \left(\int_J b(j)\,\varepsilon(j)\, dj\right)\ln S_{\text{tra}} + \left(\int_{J^*} b(j)\,\varepsilon(j)\, dj\right)\ln S_{\text{tra}}^* \\
&= \int_0^1 b(j)\ln\left(\bar{A}(j)\, b(j)\right) dj + \tilde{\theta}\ln\bar{L} + \left(1-\tilde{\theta}\right)\ln\left(\frac{\tilde{\theta}}{1-\tilde{\theta}}\bar{L}^*\right) \\
&\qquad + \left(\int_J b(j)\,\varepsilon(j)\, dj\right)\ln S_{\text{tra}} + \left(\int_{J^*} b(j)\,\varepsilon(j)\, dj\right)\ln S_{\text{tra}}^* \\
&= \ln\bar{L} + \int_0^1 b(j)\ln\left(\bar{A}(j)\, b(j)\right) dj + \left(1-\tilde{\theta}\right)\ln\tilde{\omega} \\
&\qquad + \left(\int_J b(j)\,\varepsilon(j)\, dj\right)\ln S_{\text{tra}} + \left(\int_{J^*} b(j)\,\varepsilon(j)\, dj\right)\ln S_{\text{tra}}^*,
\end{aligned}$$

where the last equality sign is obtained by using $\tilde{\theta}\bar{L}^*/\left(1-\tilde{\theta}\right) = \tilde{\omega}\bar{L}$ (from $\tilde{\theta} = \tilde{\omega}\bar{L}/\left(\omega\bar{L}+\bar{L}^*\right)$). By the definition of $\alpha(j, S, S^*)$, we can further rewrite it into

$$\begin{aligned}
U_{\text{tra}} &= \ln\bar{L} + \int_0^1 b(j)\ln\left(\bar{A}(j)\, b(j)\right) dj + \left(\int_0^1 b(j)\,\varepsilon(j)\, dj\right)\ln S_{\text{tra}} \\
&\qquad + \left(\int_{J^*} b(j)\, dj\right)\ln\tilde{\omega} + \int_{J^*} b(j)\ln\left(\frac{S_{\text{tra}}^*}{S_{\text{tra}}}\right)^{\varepsilon(j)} dj \\
&= \underbrace{\ln\bar{L} + \int_0^1 b(j)\ln\left(\bar{A}(j)\, b(j)\right) dj}_{\text{exogenous}} + \left(\int_0^1 b(j)\,\varepsilon(j)\, dj\right)\ln S_{\text{tra}} + \underbrace{\int_{J^*} b(j)\ln\left(\frac{\tilde{\omega}}{\alpha\left(j, S_{\text{tra}}, S_{\text{tra}}^*\right)}\right) dj}_{>0},
\end{aligned}$$

where the last term is positive since, for any good imported from foreign to home (i.e., for any $j \in J^*$), it necessarily holds that $\alpha\left(j, S_{\text{tra}}, S_{\text{tra}}^*\right) < \tilde{\omega}$.

On the other hand, it follows from (21) that foreign's utility level at the trade steady state satisfies

$$
\begin{aligned}
U_{\mathrm{tra}}^{*} &= \underbrace{\int_{J} b(j) \ln\left(\frac{1-\tilde{\theta}}{\tilde{\theta}} \bar{A}(j)\, S_{\mathrm{tra}}^{\varepsilon(j)} b(j)\, \bar{L}\right) dj}_{\text{utility from import goods}} + \underbrace{\int_{J^{*}} b(j) \ln\left(\bar{A}(j)\, S_{\mathrm{tra}}^{*\varepsilon(j)} b(j)\, \bar{L}^{*}\right) dj}_{\text{utility from domestically produced goods}} \\
&= \int_{0}^{1} b(j) \ln\left(\bar{A}(j)\, b(j)\right) dj + \left(\int_{J} b(j)\, dj\right) \ln\left(\frac{1-\tilde{\theta}}{\tilde{\theta}} \bar{L}\right) + \left(\int_{J^{*}} b(j)\, dj\right) \ln \bar{L}^{*} \\
&\qquad + \left(\int_{J} b(j)\, \varepsilon(j)\, dj\right) \ln S_{\mathrm{tra}} + \left(\int_{J^{*}} b(j)\, \varepsilon(j)\, dj\right) \ln S_{\mathrm{tra}}^{*} \\
&= \int_{0}^{1} b(j) \ln\left(\bar{A}(j)\, b(j)\right) dj + \tilde{\theta} \ln\left(\frac{1-\tilde{\theta}}{\tilde{\theta}} \bar{L}\right) + \left(1-\tilde{\theta}\right) \ln \bar{L}^{*} \\
&\qquad + \left(\int_{J} b(j)\, \varepsilon(j)\, dj\right) \ln S_{\mathrm{tra}} + \left(\int_{J^{*}} b(j)\, \varepsilon(j)\, dj\right) \ln S_{\mathrm{tra}}^{*} \\
&= \ln \bar{L}^{*} + \int_{0}^{1} b(j) \ln\left(\bar{A}(j)\, b(j)\right) dj - \tilde{\theta} \ln \tilde{\omega} \\
&\qquad + \left(\int_{J} b(j)\, \varepsilon(j)\, dj\right) \ln S_{\mathrm{tra}} + \left(\int_{J^{*}} b(j)\, \varepsilon(j)\, dj\right) \ln S_{\mathrm{tra}}^{*},
\end{aligned}
$$

where the last equality sign is obtained by using $\left(1-\tilde{\theta}\right) \bar{L}/\tilde{\theta} = \bar{L}^{*}/\tilde{\omega}$. Again, using the definition of $\alpha(j, S, S^{*})$ yields

$$
\begin{aligned}
U_{\mathrm{tra}}^{*} &= \ln \bar{L}^{*} + \int_{0}^{1} b(j) \ln\left(\bar{A}(j)\, b(j)\right) dj + \left(\int_{0}^{1} b(j)\, \varepsilon(j)\, dj\right) \ln S_{\mathrm{tra}}^{*} \\
&\qquad - \left(\int_{J} b(j)\, dj\right) \ln \tilde{\omega} + \left(\int_{J} b(j) \ln\left(\frac{S_{\mathrm{tra}}}{S_{\mathrm{tra}}^{*}}\right)^{\varepsilon(j)} dj\right) \\
&= \underbrace{\ln \bar{L}^{*} + \int_{0}^{1} b(j) \ln\left(\bar{A}(j)\, b(j)\right) dj}_{\text{exogenous}} + \left(\int_{0}^{1} b(j)\, \varepsilon(j)\, dj\right) \ln S_{\mathrm{tra}}^{*} + \underbrace{\int_{J} b(j) \ln\left(\frac{\alpha(j, S_{\mathrm{tra}}, S_{\mathrm{tra}}^{*})}{\tilde{\omega}}\right) dj}_{>0},
\end{aligned}
$$

where the last term is positive since, for any good imported from home to foreign (i.e., for any $j \in J$), it necessarily holds that $\alpha(j, S_{\mathrm{tra}}, S_{\mathrm{tra}}^{*}) > \tilde{\omega}$.

6.4 The properties of the $\dot{S} = 0$ and $\dot{S}^{*} = 0$ curves

We focus on the $\dot{S} = 0$ curve. A similar discussion applies to the $\dot{S}^{*} = 0$ curve (and thus is omitted here).

The slope of the $\dot{S}=0$ curve Noting that any (S, S^*) on the $\dot{S}=0$ curve satisfies $G(S) = \Omega_J(S, S^*)\bar{L}$, taking the total derivative of which yields

$$G'ds = \bar{L}\left(\frac{\partial \Omega_J}{\partial S}dS + \frac{\partial \Omega_J}{\partial S^*}dS^*\right).$$

Thus the slope of the $\dot{S}=0$ curve can be expressed by

$$\left.\frac{dS^*}{dS}\right|_{\dot{S}=0} = \frac{G' - \bar{L}\frac{\partial \Omega_J}{\partial S}}{\bar{L}\frac{\partial \Omega_J}{\partial S^*}},$$

where the stability requires that

$$G' - \bar{L}\frac{\partial \Omega_J}{\partial S} < 0.$$

The following discusses the sign of $\partial \Omega_J / \partial S^*$. Note that, using (14)

$$\Omega_J \equiv \frac{1}{\tilde{\theta}}\int_J b(j)\, e(j)\, dj == \begin{cases} \frac{1}{\tilde{\theta}}\int_k^1 b(j)\, e(j)\, dj & \text{if } S > S^* \\ \frac{1}{\tilde{\theta}}\int_0^k b(j)\, e(j)\, dj & \text{if } S < S^* \end{cases}.$$

It follows that

$$\begin{aligned} \frac{\partial \Omega_J}{\partial S^*} &= \begin{cases} \frac{1}{\tilde{\theta}}b(k)\, e(k)\left(-\frac{\partial k}{\partial S^*}\right) + \frac{\partial\left(1/\tilde{\theta}\right)}{\partial S^*}\int_k^1 b(j)\, e(j)\, dj & \text{if } S > S^* \\ \frac{1}{\tilde{\theta}}b(k)\, e(k)\frac{\partial k}{\partial S^*} + \frac{\partial\left(1/\tilde{\theta}\right)}{\partial S^*}\int_0^k b(j)\, e(j)\, dj & \text{if } S < S^* \end{cases} \\ &= \begin{cases} \frac{1}{\tilde{\theta}}b(k)\, e(k)\left(-\frac{\partial k}{\partial S^*}\right) - \frac{\partial \ln\tilde{\theta}}{\partial S^*}\Omega_J & \text{if } S > S^* \\ \frac{1}{\tilde{\theta}}b(k)\, e(k)\frac{\partial k}{\partial S^*} - \frac{\partial \ln\tilde{\theta}}{\partial S^*}\Omega_J & \text{if } S < S^* \end{cases}. \end{aligned}$$

By (16), we have

$$\tilde{\theta} = \int_J b(j)\, dj = \begin{cases} \int_k^1 b(j)\, dj & \text{if } S > S^* \\ \int_0^k b(j)\, dj & \text{if } S < S^* \end{cases},$$

which gives

$$\frac{\partial \ln\tilde{\theta}}{\partial S^*} = \begin{cases} \frac{1}{\tilde{\theta}}b(k)\left(-\frac{\partial k}{\partial S^*}\right) & \text{if } S > S^* \\ \frac{1}{\tilde{\theta}}b(k)\frac{\partial k}{\partial S^*} & \text{if } S < S^* \end{cases}.$$

Therefore,

$$\begin{aligned}\frac{\partial\Omega_J}{\partial S^*} &= \underbrace{e\left(k\right)\frac{\partial\ln\tilde{\theta}}{\partial S^*}}_{\text{extensive margin}} - \underbrace{\Omega_J\frac{\partial\ln\tilde{\theta}}{\partial S^*}}_{\text{intensive margin}} \\ &= \left(e\left(k\right)-\Omega_J\right)\frac{\partial\ln\tilde{\theta}}{\partial S^*}.\end{aligned}$$

Note that, if $e'\left(j\right)>0$,

$$\begin{cases} e\left(k\right)-\Omega_J<0 & \text{if } S>S^* \\ e\left(k\right)-\Omega_J>0 & \text{if } S<S^* \end{cases},$$

which together with $\partial\ln\tilde{\theta}/\partial S^*<0$ implies

$$\left.\frac{dS^*}{dS}\right|_{\dot{S}=0}\begin{cases} <0 & \text{if } S>S^* \\ >0 & \text{if } S<S^* \end{cases}.$$

That is, the $\dot{S}=0$ curve is downward-sloping if $S>S^*$ and upward-sloping if $S<S^*$.

On the other hand, if $e'\left(j\right)<0$,

$$\begin{cases} e\left(k\right)-\Omega_J>0 & \text{if } S>S^* \\ e\left(k\right)-\Omega_J<0 & \text{if } S<S^* \end{cases}.$$

This together with $\partial\ln\tilde{\theta}/\partial S^*<0$ implies

$$\left.\frac{dS^*}{dS}\right|_{\dot{S}=0}\begin{cases} >0 & \text{if } S>S^* \\ <0 & \text{if } S<S^* \end{cases}$$

That is, the $\dot{S}=0$ curve is upward-sloping if $S>S^*$ and downward-sloping if $S<S^*$.

The corner point of the $\dot{S}=0$ curve As $S^*\to 0$, foreign becomes "small" enough, and consequently $\tilde{\theta}\to 1$ and $k\to 0$. Noting that home produces the set of goods $J=(k,1]$, this means that home behaves like in autarky as $S^*\to 0$. As a result, the $\dot{S}=0$ curve should intersect the horizontal axis (where $S^*=0$) with $S=S_{\text{aut}}$.

As $S^*/S\to 1^-$ (i.e., approaching $S=S^*$ from below), we have that $\tilde{\omega}\to 1$ and $\tilde{\theta}\to\frac{\bar{L}}{\bar{L}+\bar{L}^*}$. Home produces the set of goods $J=(k,1]$, and the weighted average environmental intensity can be expressed by

$$\Omega_J^- = \frac{\bar{L}+\bar{L}^*}{\bar{L}} \int_k^1 b(j)\, e(j)\, dj,$$

where k satisfies that $\int_k^1 b(j)\, dj = \frac{\bar{L}}{\bar{L}+\bar{L}^*}$.

As $S^*/S \to 1^+$ (i.e., approaching $S = S^*$ from above), we have that $\tilde{\omega} \to 1$ and $\tilde{\theta} \to \frac{\bar{L}}{\bar{L}+\bar{L}^*}$. Home produces the set of goods $J = [0, k)$, and the weighted average environmental intensity can be expressed by

$$\Omega_J^+ = \frac{\bar{L}+\bar{L}^*}{\bar{L}} \int_0^k b(j)\, e(j)\, dj,$$

where k satisfies that $\int_0^k b(j)\, dj = \frac{\bar{L}}{\bar{L}+\bar{L}^*}$.

Clearly, if $e'(j) > 0$, we have

$$\Omega_J^- > \Omega_J^+,$$

meaning that the $\dot{S} = 0$ curve intersects with $S = S^*$ in different points from below and from above; the former has the level of S lower thanS_{aut} and the latter greater than S_{aut}.

In contrast, if $e'(j) < 0$, we have

$$\Omega_J^- < \Omega_J^+,$$

meaning that the $\dot{S} = 0$ curve intersects with $S = S^*$ from below with the level of S greater thanS_{aut}, and intersects from above with the level of S lower thanS_{aut}.

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