Global Value Chains and Aggregate Income Volatility

Yoichi Sugita* Taiji Furusawa† Amanda Jakobsson‡ Yohei Yamamoto§

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Abstract

This paper quantifies the general equilibrium impact of global value chains (GVC) on aggregate income volatility. Using a multi-country Ricardian model with inter-industry input-output linkages, multi-country input output tables and bilateral tariffs data, we estimate the expected level and volatility of real income per capita of individual countries and the world under counterfactual trade costs. In our benchmark case, the GVC network amplifies world-level volatility only by 1% but country-level volatility on average by 11.7%. The increase in volatility is large for poor and less populated countries.

*Graduate School of Economics, Hitotsubashi University, Japan (yoichi.sugita@r.hit-u.ac.jp)
†Faculty of Economics, University of Tokyo, Japan
‡School of Economics, Singapore Management University, Singapore
§Graduate School of Economics, Hitotsubashi University, Japan

1 Introduction

A rise in global value chains (GVC) is one of the biggest changes in manufacturing production in the last three decades. Thanks to the fall in trade and communication costs, production process has been fragmented into smaller tasks, parts and components that are produced in different countries. The increased specialization among countries has brought the aggregate income gains from trade to the world economy.

This paper analyzes the consequences of GVC on the volatility of aggregate economies. There is a growing concern that GVC might have made production riskier and more volatile. An idiosyncratic shock in one country unexpectedly affects other industries in other countries with complex input-output linkages of GVC. Studies on natural disasters found that negative shocks propagated to distant countries through global production networks (Boehm, Flaaen, and Pandalai-Nayar, 2019; Kashiwagi, Todo, and Matous, 2018).[1] As another channel, recent studies in macroeconomics emphasize that production networks can amplify micro-level idiosyncratic shocks to a large macroeconomic shock (see, e.g., Carvalho (2014) for survey on the topic). The Oil Crisis in 1970s is a case example that idiosyncratic shocks to a particular industry in a few countries had a large impact on the global economy through international input-output linkages. Furthermore, idiosyncratic shocks to industries may not offset with each other at the aggregate level since industries at different network positions could contribute to aggregate production differently as shown by Gabaix (2011), Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) and Baqaee and Farhi (2019). Since a classic paper by Long and Plosser (1983), there is a strand of literature that quantifies these interactions on the volatility of a closed economy (e.g., Horvath, 1998,2000; Foerster, Sarte, and Watson, 2011; Atalay, 2017). In the context of GVC, however, we still have limited knowledge about how GVC affects the volatility of the world economy and the distribution of volatility across individual countries.

This paper develops a framework to quantify the general equilibrium impact of GVC on aggregate income volatility based on a multi-country Ricardian model with inter-industry input-output linkages by Eaton and Kortum (2002) and Caliendo and Parro (2015).[2] Applying the idea of *struc-*

[1] Several studies also find propagation of idiosyncratic shocks through domestic supply chains (Acemoglu et al., 2016; Barrot and Sauvagnat, 2016; Carvalho et al., 2016).

[2] Caliendo, Parro, Rossi-Hansberg, and Sarte (2017) develop a multi-region Ricardian model with input-output linkages.

tural difference in difference developed by (Caliendo, Dvorkin, and Parro, 2019) and utilizing the "exact hat algebra" (Dekle, Eaton, and Kortum, 2008; Caliendo and Parro, 2015; Costinot and Rodríguez-Clare, 2014), we evaluate the causal impact of GVC on the expected level and volatility of per capita income of the world and individual countries at given point of time.

Our method proceeds in sample three steps. The first step is to identify the main driver of GVC. Our base model extends a standard model of Caliendo and Parro (2015) by incorporating for quality differentiation and the difference in trade costs between final and intermediate goods. The model's parameters are estimated for 36 countries and 31 industries in 14 years (1996–2009) using the World Input Output Database (WIOD) and UNCTAD TRAINS. The novel part of the estimation is that we identify both productivity and quality shocks at the country-industry-year-level from data on trade shares, wages and producer price indices. Our method generalizes industry productivity estimation by Levchenko and Zhang (2016) and Shikher (2012) by allowing quality shocks. Counterfactual exercises of the model shows that the decline in trade costs was the main driver of GVC, which is consistent with a previous finding by Johnson and Noguera (2017) from a Armington-based structural gravity model.

The second step is to estimate the stochastic process of idiosyncratic productivity and quality shocks. Using three-level factor models, we decompose those shocks into global-level factors (common shocks), country-level factors, industry-level factors and idiosyncratic shocks. Idiosyncratic shocks play substantial roles and account for more than 30% of the total variances in shocks. From the estimated process of idiosyncratic shocks, we simulate 100 samples of idiosyncratic shocks.

The final step is to calculate counterfactuals changes in variables for 100 samples of idiosyncratic shocks under different scenarios with respect to hypothetical trade costs. In our benchmark case, GVC increased the mean and volatility of world real per capita income by 4.5% and 1.5%. At the individual country-level, GVC increased on average the mean and volatility of real per capita income by 8% and 11%, respectively. We also find that the increase in volatility is large for poor and less populated countries.

Related Literature (Incomplete) Our study is related to several literature beside the ones already mentioned above. We will discuss the related literature in a future version.

There is a long literature on county's trade openness and volatility. A study close to our study is Caselli, Koren, Lisicky, and Tenreyro (2015) who quantify the effect of trade on volatility in the Eaton-Kortum model. They analyze the overall effect of international trade without distinguishing trade in intermediate goods and final goods. We quantify a particular channel of trade on volatility, the network aggregation of idiosyncratic shocks through GVC.

2 Methodology

2.1 Model

The model is a static Ricardian model with N countries indexed by $i, n \in \{1, ..., N\}$, S industries indexed by $s, k \in \{1, ..., S\}$ and one homogenous factor, labor. All goods and labor are traded in perfectly competitive markets. There is no saving or investment.

Each industry produces two types of goods with different usages, final goods and intermediate goods. Final goods, denoted by f, are used only for final consumption, while intermediate goods, denoted by m, are used only for inputs for production. Each usage $u \in \{f, m\}$ in industry s consists of a continuum of varieties $\omega^{su} \in [0, 1]$.

Country i's representative consumer's utility function:

$$U_n = \prod_{s=1}^{S} \left(Q_{nt}^{sf}\right)^{\alpha_n^s}, \; Q_{nt}^{sf} \equiv \left[\int_0^1 q_{nt}^{sf*}\left(\omega^{sf}\right)^{\frac{\sigma^{sf}-1}{\sigma^{sf}}} d\omega^{sf}\right]^{\frac{\sigma^{sf}}{\sigma^{sf}-1}},$$

where $\sigma^{sf} > 0$ is the elasticity of substitution and $q_{nt}^{sf*}\left(\omega^{sf}\right)$ is country n's quality-adjusted consumption of variety ω^{sf} at time t, which is given by

$$q_{nt}^{sf*}\left(\omega^{sf}\right) \equiv \sum_{i=1}^{n} \kappa_{it}^{s} q_{nit}^{sf}\left(\omega^{sf}\right), \; \prod_{i=1}^{N} \left(\kappa_{it}^{s}\right)^{1/N} = 1,$$

where $q_{nit}^{sf}\left(\omega^{sf}\right)$ is country n's consumption of variety ω^{sf} produced in country i at time t and κ_{it}^{s} is a quality parameter shared by all varieties of both usages within industry s in country i at time t. The quality parameter is normalized so that the world average quality satisfies $\prod_{i=1}^{N}\left(\kappa_{it}^{s}\right)^{1/N} = 1$. This normalization is imposed because multiplying a positive scalar to κ_{it}^{s} for all i does not change each country's expenditure in industry s.

A firm in industry s in country n produces $y_{nt}\left(\omega^{su}\right)$ units of variety ω^{su} of usage u by the following constant returns to scale production function:

$$y_{nt}\left(\omega^{su}\right) = A^s_{nt} z_n\left(\omega^{su}\right) l_{nt}\left(\omega^{su}\right)^{\beta^s_n} \prod_{k=1}^{S} m^{sk}_{nt}\left(\omega^{su}\right)^{\beta^{sk}_n},\ \beta^s_n + \sum_{k=1}^{S} \beta^{sk}_n = 1,$$

where $l_{nt}\left(\omega^{su}\right)$ is labor input, $A^s_{nt} z_n\left(\omega^{su}\right)$ is total factor productivity (TFP). A^s_{nt} is the country-industry specific component of TFP and $z_n\left(\omega^{su}\right)$ is the idiosyncratic component drawn from a Frichet distribution $F^s\left(z\right) = \exp\left(-z^{-\theta^s}\right)$. $m^{sk}_{nt}\left(\omega^{su}\right)$ is the intermediate input index of good k given by

$$m^{sk}_{nt}\left(\omega^{su}\right) \equiv \left[\int_0^1 \tilde{m}^{*sk}_{nt}\left(\omega^{km};\omega^{su}\right)^{\frac{\sigma^{km}-1}{\sigma^{km}}} d\omega^{km}\right]^{\frac{\sigma^{km}}{\sigma^{km}-1}},$$

where $\sigma^{km} > 0$ is the elasticity of substitution and $\tilde{m}^{*sk}_{it}\left(\omega^{km};\omega^{su}\right)$ is the quality-adjusted input of variety ω^{km} for production of variety ω^{su} in country i, which is given by

$$\tilde{m}^{*sk}_{nt}\left(\omega^{km};\omega^{su}\right) \equiv \sum_{i=1}^{N} \kappa^k_i \tilde{m}^{sk}_{nit}\left(\omega^{km};\omega^{su}\right),$$

where $\tilde{m}^{sk}_{nit}\left(\omega^{km};\omega^{su}\right)$ is the amount of ω^{km} produced in country i and used as input for production of ω^{su} in country n.

Country n purchases variety ω^{sf} with the lowest quality adjusted price with $p^*_{nt}\left(\omega^{su}\right) \equiv \min_{i=1}^{N} \frac{p_{nit}\left(\omega^{sf}\right)}{\kappa^s_i}$, where $p_{nit}(\omega^{su})$ is the unit cost of supplying from country i to country n. The quality-adjusted price index for usage u of industry s in country i is given by $P^{su*}_{nt} = \left[\int_0^1 p^*_{it}\left(\omega^{su}\right)^{1-\sigma^{su}} d\omega^{su}\right]^{\frac{1}{1-\sigma^{su}}}$. Trade costs is of iceberg type $p_{nit}(\omega^{su}) \equiv d^{su}_{nit} p_{iit}(\omega^{su})$ and consist of tariffs τ^s_{nit} and non-tariff barriers D^{su}_{nit} as

$$d^{su}_{nit} = (1 + \tau^s_{nit}) D^{su}_{nit}, \tag{1}$$

where the triangle inequality $d^{su}_{njt} d^{su}_{jit} \geq d^{su}_{nit}$ is satisfied and each component of domestic trade costs is normalized to one: $d^{su}_{iit} = 1 + \tau^s_{iit} = D^{su}_{iit} = 1$.

Note that within an industry, intermediate goods and final goods share the same quality parameter, productivity parameter and Frichet parameter. The only meaningful distinction between final usage and intermediate usage is trade costs.

2.2 Equilibrium in Changes

The unit cost of producing variety ω^{su} in country i is $p_{iit}(\omega^{su}) = \frac{\xi_i^s c_{it}^s}{A_{it}^s z_i(\omega^{su})}$ where ξ_i^s is constant, c_{it}^s is the unit cost index given by

$$c_{it}^s = w_{it}^{\beta_i^s} \prod_{k=1}^{S} \left(P_{it}^{km*}\right)^{\beta_i^{sk}}, \tag{2}$$

where w_{it} is wage in country i. Let $\Lambda_{it}^s \equiv \left(\frac{\kappa_{it}^s A_{it}^s}{\xi_i^s}\right)^{\theta^s}$ be the combined positive shock of quality and productivity shocks. From the standard mathematic of the Eaton and Kortum model, the quality-adjusted price index for usage u of industry s in country i is given by

$$\left(\frac{P_{nt}^{su*}}{\gamma^{su}}\right)^{-\theta^s} = \sum_{i=1}^{N} \Lambda_{it}^s \left(c_{it}^s d_{nit}^{su}\right)^{-\theta^s} \equiv \Phi_{nt}^{su} \tag{3}$$

where $\gamma^{su} \equiv \left[\Gamma\left(\frac{\theta^s+1-\sigma^{su}}{\theta^s}\right)\right]^{1/(1-\sigma^{su})}$ and Γ is the gamma function. The trade share of country i's products with usage u in industry s in market n is given by

$$\pi_{nit}^{su} = \frac{\Lambda_{it}^s \left(c_{it}^s d_{nit}^{su}\right)^{-\theta^s}}{\Phi_{nt}^{su}}. \tag{4}$$

Let X_{nt}^{su} be country n's tariff-inclusive expenditure on usage u in industry s. The Cobb-Douglass production and utility functions imply

$$X_{nt}^{sm} = \sum_{k=1}^{S} \beta_n^{\beta ks} Y_{nt}^k \text{ and } X_{nt}^{sf} = \alpha_n^s \left[w_{nt}L_{nt} + R_{nt} + TD_{nt}\right] \tag{5}$$

where $Y_{nt}^k = \sum_{i=1}^N \frac{\pi_{int}^{kf}}{1+\tau_{int}^k} X_{it}^{kf} + \sum_{i=1}^N \frac{\pi_{int}^{km}}{1+\tau_{int}^k} X_{it}^{km}$ is the tariff-exclusive gross revenue of industry k, $R_{nt} = \sum_{s=1}^S \sum_{i=1}^N \frac{\tau_{nit}^s}{1+\tau_{nit}^s} (\pi_{nit}^{sf} X_{nt}^{sf} + \pi_{nit}^{sm} X_{nt}^{sm})$ is tariff revenue and TD_{nt} is country n's trade deficit given by

$$TD_{nt} = \sum_{s=1}^{S} \sum_{i=1}^{N} \left(\frac{\pi_{nit}^{sf} X_{nt}^{sf} + \pi_{nit}^{sm} X_{nt}^{sm}}{1+\tau_{nit}^s} - \frac{\pi_{int}^{sf} X_{it}^{sf} + \pi_{int}^{sm} X_{it}^{sm}}{1+\tau_{int}^s} \right). \tag{6}$$

Following Dekle et al. (2008) and Caliendo and Parro (2015), TD_{nt} is exogenously given. Conditions (2), (3), (4), (5) and (6) determine an equilibrium.

Following Dekle et al. (2008) and Caliendo and Parro (2015), it is convenient for considering a system of equilibrium conditions for changes in variables. Let x_0 be the value of variable x in an initial equilibrium at time t, x'_t be its value in a counterfactual equilibrium at time t, and $\hat{x}_t \equiv x'_t/x_0$ be the counterfactual change of variable x. In the analysis below, an initial equilibrium will often be either (i) data of time t variables $x_0 = x_t$ or (ii) data of time $t-1$ variables $x_0 = x_{t-1}$. As an exogenous constraint on changes in trade deficit, we assume trade deficit relative to the world GDP remains the same between two equilibriums. Then, we obtain equilibrium conditions for variable changes as follows.

Definition 1. A collection of changes in endogenous variables $\left\{\hat{w}_{it}, \hat{c}^s_{it}, \hat{P}^{su*}_{it}, \hat{\pi}^{su}_{int}, \hat{X}^{su}_{nt}\right\}$ satisfy the following conditions:

$$\hat{c}^s_{it} = \hat{w}_{it}^{\beta^s_i} \prod_{k=1}^{S} \left(\hat{P}^{km*}_{it}\right)^{\beta^{sk}_i} \tag{7}$$

$$\left(\hat{P}^{su*}_{it}\right)^{-\theta^s} = \sum_{h=1}^{N} \pi^{su}_{nh0} \hat{\Lambda}^s_{ht} \left(\hat{c}^s_{ht} \hat{d}^{su}_{nht}\right)^{-\theta^s} \tag{8}$$

$$\hat{\pi}^{su}_{nit} = \frac{\hat{\Lambda}^s_{ht} \left(\hat{c}^s_{ht} \hat{d}^{su}_{nht}\right)^{-\theta^s}}{\left(\hat{P}^{su*}_{it}\right)^{-\theta^s}} \tag{9}$$

$$X^{sf\prime}_{nt} = \alpha^s_n \left[\hat{w}_{nt} \hat{L}_{nt} w_{n0} L_{n0} + \sum_{s=1}^{S} \sum_{i=1}^{N} \frac{\tau^{s\prime}_{nit}}{1+\tau^{s\prime}_{nit}} \left(\pi^{sf\prime}_{nit} X^{sf\prime}_{nt} + \pi^{sm\prime}_{nit} X^{sm\prime}_{nt}\right) + TD'_{nt}\right] \tag{10}$$

$$X^{sm\prime}_{nt} = \sum_{k=1}^{S} \beta^{ks}_n \left(\sum_{i=1}^{N} \frac{\pi^{kf\prime}_{int}}{1+\tau^{k\prime}_{int}} X^{kf\prime}_{it} + \sum_{i=1}^{N} \frac{\pi^{km\prime}_{int}}{1+\tau^{k\prime}_{int}} X^{km\prime}_{it}\right) \tag{11}$$

$$TD'_{nt} = \sum_{s=1}^{S} \sum_{i=1}^{N} \left(\frac{\pi^{sf\prime}_{nit} X^{sf\prime}_{nt} + \pi^{sm\prime}_{nit} X^{sm\prime}_{nt}}{1+\tau^{s\prime}_{nit}} - \frac{\pi^{sf\prime}_{int} X^{sf\prime}_{it} + \pi^{sm\prime}_{int} X^{sm\prime}_{it}}{1+\tau^{s\prime}_{int}}\right). \tag{12}$$

$$\frac{TD'_{nt}}{\sum_i \hat{w}_{it} \hat{L}_{it} w_{i0} L_{i0}} = \frac{TD_{n0}}{\sum_i w_{i0} L_{i0}} \tag{13}$$

Computation of counterfactuals follows the algorithm developed by Caliendo and Parro (2015) that solves the above system for wage changes. Since $\sum_{n=1}^{N} TD_{nt} = 0$ from the Warlas's law, there are only $N-1$ independent equations of (13). Therefore, we normalize $\hat{w}_N = 1$. In Appendix, we express the equilibrium conditions and the algorithm in matrix to facilitate computation.

2.3 Structural Difference in Difference Analysis

Our goal is to quantify the causal effect of GVC on the risk of country's real income per capita . For simplicity, we proxy country i's real income per capita at time t by real wage $W_{it} \equiv w_{it}/\prod_s \left(P^{sf*}_{it}\right)^{\alpha^s_i}$

in the model, though the following method can incorporate additional income sources such as tariff revenue. We let the model to generate counterfactuals indexed by (d, r). The first index d is a binary indicator $d \in \{0, 1\}$ on the existence of GVC ($d = 1$) or not ($d = 0$), which corresponds to a treatment indicator in a usual difference-in-difference analysis. The second index r indicates a state of nature regarding the realization of idiosyncratic shocks.

We evaluate the risk of real wage by the first moment, mean, and the second moment, standard deviation. Denote $W_{it}(d, r)$ be the real wage of country i at time t in a counterfactual equilibrium indexed by (d, r). Our goal is to obtain the impacts of GVC on the mean and standard deviation of country i's real wage at time t:

$$\Delta MW_{it} \equiv E_r\left[\ln W_{it}(1, r)\right] - E_r\left[\ln W_{it}(0, r)\right] \text{ and } \Delta VW_{it} \equiv \sqrt{Var_r\left[\ln W_{it}(1, r)\right]} - \sqrt{Var_r\left[\ln W_{it}(0, r)\right]}, \tag{14}$$

where E_r and Var_r are the expectation and variance operators with respect to r, respectively.

To directly calculate (14) is challenging because we need to estimate a number of parameters to calculate counterfactual levels of endogenous variables. The idea of *structural difference-in-difference* by Caliendo et al. (2019) greatly simplifies the problem of estimating (14). The exact hat algebra with $\hat{W}_{it}(1, r) = W_{it}(1, r)/W_{it}$ rewrite (14) as

$$\Delta MW_{it} = E_r\left[\ln \hat{W}_{it}(1, r)\right] - E_r\left[\ln \hat{W}_{it}(0, r)\right] \text{ and } \Delta VW_{it} = \sqrt{Var_r\left[\ln \hat{W}_{it}(1, r)\right]} - \sqrt{Var_r\left[\ln \hat{W}_{it}(0, r)\right]}. \tag{15}$$

Therefore, the first and second conditional moments of $\ln \hat{W}_{it}(d, r)$ are sufficient for obtaining (14).

We calculate $\hat{W}_{it}(d, r)$ as follows. Suppose we have R random samples of the combined shocks $\left\{\left(\hat{\Lambda}^s_{it}(r)\right)_{i=1}^N\right\}_{i=1}^R$. Suppose also that we know the model's deep parameter d determining the extent of GVC. Then, we calculate $2R$ counterfactual equilibrium for all combinations of (d, r) and obtain $2R$ real wage changes of country n at time t, $\left\{\hat{W}_{it}(1, r), \hat{W}_{it}(0, r)\right\}_{r=1}^R$. From them, we calculate the sample analogue of ΔMW_{it} and ΔVW_{it} by the sample means and standard deviation of $\ln \hat{W}_{it}(1, r)$ and $\ln \hat{W}_{it}(0, r)$.

We also calculate the impact of GVC on the risk of the world income. Define the world real wage by a geometric mean weighted by worker shares, $W_{wt} \equiv \prod_{i=1}^N (W_{it})^{s^L_{it}}$ where $s^L_{it} \equiv L_{it}/\sum_i L_{it}$. Since $\ln \hat{W}_{wt}(r, d) = \sum_i s^L_{it} \ln \hat{W}_{it}(r, d)$, ΔMW_{wt} and ΔVW_{wt} can be obtained by a similar method for obtaining (15).

To implement the above method, we need to obtain two parameters. First, we obtain random samples of idiosyncratic shocks by estimating the probability distribution of productivity and quality shocks. Second, we identify the model's deep parameters that drove the rise in GVC.

3 Estimation of Parameters

3.1 Data

The main dataset is WIOD (the World Input Output Database) of 2013 release, which covers 35 ISIC industries (3 digit level) and 40 countries for every year from 1995 to 2011. Because of differences in industry classification across countries, some industries in some countries have missing values. Following Costinot and Rodríguez-Clare (2014), we combine WIOD industries 4 and 5 into WIOD industries 4, WIOD industry 19 and 20 into WIOD industry 19, and WIOD industries 31, 34, and 35 into WIOD industries 31. Five countries (Cyprus, Indonesia, Luxembourg, Latvia, and Malta) and two years (2010 and 2011) are dropped because of data availability. The final dataset ends up with 31 industries and 34 countries plus the rest of the world (RoW) for 1995–2009. Countries except RoW account for 88% of the world GDP in 2000. Usage is assigned based on the demand side: expenditures by the 31 good/service producing industries are classified as intermediate goods expenditure, while expenditures by other categories are as final goods expenditure. All service industries are treated non-tradable goods since the statistic of international trade in services is likely to be heterogenous in data quality across countries and years.

The data source for tariffs is simple average MFN tariffs and simple average preferential tariffs in UNCTAD TRAINS downloaded from the World Trade Integrated System. Tariffs reported at the Harmonized System 6 digit level are aggregated to the WIOD industry level, using fixed weights of import volume in 1995. We also imputed for missing years up to +/- 3 years.

The bilateral tariffs τ_{nit}^{s} are obtained for around 30 % of bilateral country-industry-year combinations. As an alternative tariff measure, we also construct *quasi bilateral tariffs* $\tilde{\tau}_{nit}^{s} \equiv (1 - PTA_{nit})\,\tau_{nt}^{MFN,s}$ where $\tau_{nt}^{MFN,s}$ is MFN tariffs by country s in year t and PTA_{nit} is an indicator on whether countries i and n had a free trade agreement or formed a customs union in year t. The correlation between τ_{nit}^{s} and $\tilde{\tau}_{nit}^{s}$ (for $n \neq i$) is 0.913. Though $\tilde{\tau}_{nit}^{s}$ can be calculated for all country-industry-year combinations, it is of course subject to measurement errors because not all tariffs are zero in typical preferential trade agreements. We will address the issue of measurement errors below.

3.2 Calibration and Estimation

Trade Shares, Wages and Cobb-Douglass Parameters WIOD reports bilateral final trade values of good s that country n purchased from country i, M_{nit}^{sf}, and bilateral intermediate goods trade values of good s that industry r in country n purchased from country i, M_{nit}^{rs}. Since those values are recorded in producer prices where tariffs are excluded, we construct tariff-inclusive expenditures as $X_{nt}^{sf} \equiv \sum_{i=1}^{N} M_{nit}^{sf}(1+\tilde{\tau}_{nit}^{s})$ and $X_{nt}^{sm} = \sum_{i=1}^{N}\sum_{r=1}^{S} M_{nit}^{rs}(1+\tilde{\tau}_{nit}^{s})$. The gross total sales of industry s in country n in producer prices are $Y_{nt}^{s} = \sum_{i=1}^{N}\left(M_{int}^{sf} + \sum_{r=1}^{S} M_{int}^{rs}\right)$. The value added of industry s in country n is $V_{nt}^{s} = Y_{nt}^{s} - \sum_{r=1}^{S} M_{nit}^{rs}(1+\tilde{\tau}_{nit}^{s})$ and the total value-added (GDP) of country n is $V_{nt} \equiv \sum_{s=1}^{S} V_{nt}^{s}$.

The labor endowment is obtained by the total number of workers, $L_{nt} \equiv \sum_{s=1}^{S} L_{nt}^{s}$. Interpreting labor as a composite factor that receives all value-added as labor income, the wage is obtained by GDP per worker $w_{nt} = V_{nt}/L_{nt}$. The Cobb-Douglass parameters are country-specific and stable over time, which are obtained as

$$\alpha_n^s = \frac{\sum_t X_{nt}^{sf}}{\sum_t \sum_{s=1}^{S} X_{nt}^{sf}},\ \beta_n^s = \frac{\sum_t V_{nt}^s}{\sum_t Y_{nt}^s},\ \text{ and } \beta_n^{sk} = \frac{\sum_t M_{nit}^{sk}(1+\tilde{\tau}_{nit}^{k})}{\sum_t Y_{nt}^s}.$$

Frechet Parameter θ^s We estimate Frechet parameters θ^s, also called as trade elasticities, by exploiting variations of bilateral tariffs in the gravity model, following the spirit of Caliendo and Parro (2015). Trade costs d_{nit}^{su} are modeled as:

$$\ln d_{nit}^{su} = \ln\left(1+\tau_{nit}^{s}\right) + \sum_{k} TC_{ni,k}\delta_{kt}^{su} + \varepsilon_{nit}^{su}, \tag{16}$$

where τ_{nit}^{su} is bilateral tariff rates, ε_{nit}^{su} is idiosyncratic trade costs, and $TC_{ni,k}$ is k-th variable representing country-pair characteristics, which may have different impacts across time and across usages. As $TC_{ni,k}$, we include log of distance, contiguity dummy, common language dummy, ever-colonial relationship dummy, a dummy indicating international trade, which are taken from taken from CEPII datasets. Substituting (16) into (4), we estimate the following fixed effect gravity model:

$$\ln \pi_{nit}^{su} = -\theta^{s} \ln\left(1+\tau_{nit}^{s}\right) + \sum_{t}\sum_{k} TC_{ni,k} I_{\{Year=t\}} \left(\beta_{kt}^{f} + I_{\{u=m\}}\beta_{kt}^{m}\right) + ex_{it}^{s} + im_{nt}^{su} + \varepsilon_{nit}^{su} \quad (17)$$

where $I_{\{Year=t\}}$ is a year dummy, $I_{\{u=m\}}$ is an indicator of trade in intermediate goods, ex_{it}^{s} is time-exporter fixed effects and im_{it}^{su} is usage-time-importer fixed effects, respectively.

We estimate (17) for each WIOD industry (tradable goods industries from 1 to 16) separately, pooling years from 1995 to 2011 and including observations where preferential tariffs are observable.[3] Table 1 reports the estimated trade elasticities. They are all precisely estimated with small standard errors and the size of the estimates are reasonable given that the existence of an equilibrium requires $\theta^{s} > \max\left\{\sigma^{sm}, \sigma^{sf}\right\} - 1$. As common for existing approaches to estimating θ^{s} in the Eaton-Kortum model, the current approach cannot be applied for non-tradable industries. For non-traded service industries, we set $\theta^{s} = 7.31$ from the median estimate among tradable industries.

Trade Costs and Fixed Effects We need estimate bilateral trade costs and exporter fixed effects in the gravity equation like (17) for all combinations of country pairs, tradable industries and years. Since our bilateral tariff data covers only 30% of the combinations, we use quasi bilateral tariffs $\tilde{\tau}_{nit}^{s}$ to obtain trade costs and exporter fixed effects. A challenge is that directly estimating (17) with $\tilde{\tau}_{nit}^{s}$ would suffer from the endogeneity due to the measurement errors of $\tilde{\tau}_{nit}^{s}$ in the right hand side. To avoid this pitfall, we pursue alternative procedures.

We first obtain trade costs d_{nit}^{su} by a modified Head-Ries index incorporating for asymmetric bilateral tariffs. Assume that non-tariff trade costs are symmetric in direction $D_{nit}^{su} = D_{int}^{su}$ and normalized as $D_{iit}^{su} = 1$ for domestic trade. For $\pi_{nit}^{su} > 0$ and $\pi_{int}^{su} > 0$, trade costs (1) and trade shares (4) imply

$$\ln \frac{\pi_{nit}^{su}\pi_{int}^{su}}{\pi_{nnt}^{su}\pi_{iit}^{su}} = -\theta^{s} \ln\left(1+\tau_{nit}^{s}\right)\left(1+\tau_{int}^{s}\right) - 2\theta^{s} \ln D_{nit}^{su}$$

[3] A potential concern in estimating (17) is those bilateral trades showing zero trade volume. Those zero bilateral trade are dropped when estimating (17). As a robustness check, we estimate by the poisson pseudo maximum likelihood (PPML) for two samples: the one including zero trade volume and the one not including. Although PPML find slightly greater θ than OLS, PPML estimates of the two samples are almost identical, which means that the main difference between PPML and OLS comes from the difference in the estimation method, not from dropping zeros. OLS estimation also allows the gravity error term to include unobserved trade costs as in (16) and (17), but PPML estimation does not. Therefore, we use OLS estimates as our benchmark.

Table 1: Trade Elasticities (Frechet Parameters)

WIOD	Industry Description	Theta	Robust SE	n.obs
1	Agriculture, Hunting, Forestry and Fishing	6.26***	(0.54)	36,980
2	Mining and Quarrying	8.05***	(1.60)	33,654
3	Food, Beverages and Tobacco	7.31***	(0.39)	37,101
4	Textile Products, Leather Products and Footwear	6.31***	(0.32)	37,467
6	Wood and Products of Wood and Cork	9.12***	(0.60)	37,133
7	Pulp, Paper, Paper, Printing and Publishing	11.37***	(0.71)	37,394
8	Coke, Refined Petroleum and Nuclear Fuel	6.10***	(0.95)	36,633
9	Chemicals and Chemical Products	6.31***	(0.54)	37,470
10	Rubber and Plastics	6.22***	(0.41)	37,433
11	Other Non-Metallic Mineral	4.78***	(0.47)	37,391
12	Basic Metals and Fabricated Metal	7.78***	(0.54)	37,446
13	Machinery, Nec	7.43***	(0.46)	37,480
14	Electrical and Optical Equipment	9.69***	(0.78)	37,166
15	Transport Equipment	7.13***	(0.40)	36,946
16	Manufacturing, Nec; Recycling	8.01***	(0.52)	37,438

***: 1% significance

Note: Table shows the estimates of Frichet parameters in column "Theta" with robust standard errors for tradable industries in WIOD (WIOD industries 4 and 5 are merged into industry 4).

Using $\tilde{\tau}_{nit}^{s}$, we estimate non-tariff components for all combinations of country pairs, tradable industries and years with positive trade flows

$$\ln D_{nit}^{su} = \frac{1}{2\theta^{s}} \ln \frac{\pi_{nit}^{su}\pi_{int}^{su}}{\pi_{nnt}^{su}\pi_{iit}^{su}} - \frac{1}{2} \ln\left(1+\tilde{\tau}_{nit}^{s}\right) - \frac{1}{2} \ln\left(1+\tilde{\tau}_{int}^{s}\right).$$

Then, trade costs are obtained as

$$\ln d_{nit}^{su} = \frac{1}{2} \ln\left(\frac{1+\tilde{\tau}_{nit}^{s}}{1+\tilde{\tau}_{int}^{s}}\right) + \frac{1}{2\theta^{s}} \ln \frac{\pi_{nit}^{su}\pi_{int}^{su}}{\pi_{nnt}^{su}\pi_{iit}^{su}}. \tag{18}$$

Next we estimate the following regression with exporter fixed effects and importer fixed effects

$$\ln \pi_{nit}^{su} - \theta^{s} \ln d_{nit}^{su} = ex_{it}^{s} + im_{nt}^{su} + \varepsilon_{nit} \tag{19}$$

by OLS for each year separately. We drop an exporter dummy for a benchmark country b when we estimate (19).[4] Then, from (4), exporter fixed effects and importer-usage fixed effects estimate

$$\hat{ex}_{it}^{s} = \ln S_{it} - \ln S_{bt} \text{ and } \hat{im}_{nt}^{su} = \ln S_{bt} - \ln \Phi_{nt}^{su}, \tag{20}$$

where $S_{it} \equiv \Lambda_{it}^{s}\left(c_{it}^{s}\right)^{-\theta^{s}}$ is the competitiveness index of country i's industry s.

The above procedure aims to reduce the influence from the measurement error in quasi tariffs $\varepsilon_{nit}^{s\tau} \equiv \ln\left(1+\tilde{\tau}_{nit}^{s}\right) - \ln\left(1+\tau_{nit}^{s}\right)$. Let d_{nit}^{su*} be bilateral trade costs calculated with τ_{nit}^{su} in data. Then, the measurement errors in trade costs is $\ln d_{nit}^{su} - \ln d_{nit}^{su*} = \left(\varepsilon_{nit}^{s\tau} - \varepsilon_{int}^{s\tau}\right)/2$. In regression (19), the measurement error appears in the left hand side. Thus, the error term in (19) absorbs the measurement error as $\varepsilon_{nit} = \theta^{s}\left(\varepsilon_{int}^{\tau} - \varepsilon_{nit}^{\tau}\right)/2$. One potential source of the measurement error is gradual liberalization. While quasi bilateral tariffs assume zero tariff for trade between countries signing a preferential trade agreement, it is often the case that actual preferential tariffs are gradually reduced overtime, which implies $\varepsilon_{int}^{s\tau} < 0$ and $\varepsilon_{nit}^{s\tau} < 0$. In the above approach, the measurement errors $\varepsilon_{int}^{s\tau} < 0$ and $\varepsilon_{nit}^{s\tau} < 0$ offset with each other to reduce the effect of the measurement errors on the estimation of trade costs (18) and fixed effects (19).

Table 2 summarizes the estimated trade costs. Panel A presents summary statistics of trade

[4] We must exclude one dummy from exporter dummies and importer dummies because the sum of all exporter dummies equals to the sum of all importer dummies.

costs, tariffs and non-tariff barriers (NTB) in ad valorem equivalent rates, which are $d^s_{nit}-1$, $\tilde{\tau}^s_{ijt}$ and $D^s_{nit}-1$, respectively, for 1995 and 2007. Year 2007 is chosen to avoid the influence of the Lehman crisis and the great trade collapse. Two patterns can be seen. First, by 1995, tariffs were already low with mean 7.6% and account for only a minor share in overall trade costs with mean 187%. Second, both tariffs and NTB had dropped significantly over time. Panel B report the regressions of log trade costs $\ln d^s_{nit}$, log quasi tariffs $\ln(1+\tilde{\tau}^s_{nit})$ and log NTB $\ln D^s_{nit}$ on the number of years from 1995 with industry fixed effects, exporter fixed effects and importer fixed effects for 1995–2011 in columns (1), (3) and (4), respectively. By construction, the coefficients in (1) equals the sum of those in (3) and (4). Column (1) shows that average trade costs declined by 0.7 percent per year. Columns (3) and (4) show that tariffs and NTB equally contributed for the decline in total trade costs. Columns (2) and (5) investigate the difference between final goods and intermediate goods by including the dummy of intermediate goods and its interaction with the number of years from 1995. The coefficient of the intermediate good dummy indicates that in 1995, trade costs of final goods were 2% higher than trade costs of intermediate goods. After 1995, trade costs declined annually by 0.8% for final goods and by 0.6% for intermediate goods.

Industry Shocks

From (2) and the definition of the competitiveness index $S^s_{it} \equiv \Lambda^s_{it}\left(c^s_{it}\right)^{-\theta^s}$, the change in the combined shocks is expressed as the sum of competitiveness and unit costs changes:

$$d\ln\Lambda_{it} = d\ln S_{it} + d\ln W^{\beta\theta}_{it} + B_i d\ln P^{m*\theta}_{it} \tag{21}$$

where $d\ln S_{it} = (d\ln S^1_{it}, ..., d\ln S^S_{it})^T$, $d\ln\Lambda_{it} = (d\ln\Lambda^1_{it}, ..., d\ln\Lambda^S_{it})^T$, $d\ln W^{\beta\theta}_{it} = (\theta_1\beta^1_i d\ln w_{it}, ..., \theta_S\beta^S_i d\ln w_{it})^T$ and $d\ln P^{m*\theta}_{it} = (\theta_1 d\ln P^{1m*}_{it}, ..., \theta_S d\ln P^{Sm*}_{it})^T$ are $S\times 1$ vectors and B_i is a $S\times S$ input-output matrix with β^{sk}_i as its sk element. From (4) and (8), the change in a price index change is obtained as

$$d\ln P^{m*\theta}_{it} = -d\ln\Phi^m_{it} = d\ln\pi^{sm}_{iit} - d\ln S_{it}.$$

Then, combined shocks are expressed as

$$d\ln\Lambda_{it} = (I-B_i)d\ln S_{it} + d\ln W^{\beta\theta}_{it} + B_i d\ln\pi^m_{iit}. \tag{22}$$

Table 2: Trade Costs

Panel A: Summary Statistics

	Year	Mean	SD	Min	P25	Median	P75	Max	N
Trade Costs (AVE)	1995	1.877	1.643	-0.14	0.934	1.448	2.289	57.37	34,170
	2007	1.551	1.298	-0.582	0.757	1.194	1.898	18.183	34,896
Quasi Tariff Rate	1995	0.076	0.095	0	0.01	0.051	0.106	0.749	17,358
	2007	0.028	0.06	0	0	0	0.033	0.585	17,790
NTB (AVE)	1995	1.67	1.501	-0.14	0.821	1.271	2.032	51.54	34,170
	2007	1.477	1.232	-0.587	0.728	1.131	1.802	15.8	34,896

Panel B: Time Trend: 1995–2011

Depenent Variables	Log Trade Costs		Log (1+ Quasi Tariffs)	Log NTB	
	(1)	(2)	(3)	(4)	(5)
Year	-0.0070***	-0.0082***	-0.0034***	-0.0036***	-0.0047***
(from 1995)	(0.0008)	(0.0008)	(0.0001)	(0.0007)	(0.0008)
Usage=m		-0.0223***			-0.0220***
		(0.0018)			(0.0018)
Year x Usage=m		0.0023***			0.0023***
		(0.0003)			(0.0003)
Industry FE	X	X	X	X	X
Importer FE	X	X	X	X	X
Exporter FE	X	X	X	X	X
Observations	590,712	590,712	590,712	590,712	590,712
R^2	0.638	0.638	0.568	0.626	0.626

Note: SE clustered at the year level; Significance: *** 1%

The above derivation used a similar technique in Shikher (2012) and Levchenko and Zhang (2016) where the authors calculated relative productivity $\ln A_{it}^s - \ln A_{bt}^s$ to a benchmark country. We relax their assumption that input-output matrices and quality parameters are common across countries and time, $B_i = B$ and $\kappa_i^s = \kappa^s$ and obtain productivity changes $d\ln A_{it}^s$ and quality changes $d\ln\kappa_{it}^s$, separately.

We obtain $d\ln S_{it}$, using data on price deflators and exporter fixed effects in the gravity equation. The WIOD socio economic accounts dataset reports price deflators for gross industrial outputs, which are usually indexes of producer (factory-gate) prices of domestically produced products between t and $t-1$. Consider variety ω that country i produces at both t and $t-1$. The log price change of the variety ω between t and $t-1$ is $d\ln p_{iit}^s(\omega) = d\ln c_{it}^s - d\ln A_{it}^s$. We assume that the statistical offices of countries create price deflators for industrial outputs $\frac{\tilde{P}_{it}^s}{\tilde{P}_{it-1}^s}$ by sampling only prices of goods domestically produced at both t and $t-1$ and by aggregating them with time-invariant weights, which is roughly consistent with the best practice recommended by international organizations (IMF, 2004). Under this assumption, the price deflators reflect changes in unit production costs as:

$$d\ln\tilde{P}_{it}^s = d\ln c_{it}^s - d\ln A_{it}^s.$$

From the definitions of $S_{it}^s \equiv \Lambda_{it}^s\left(c_{it}^s\right)^{-\theta^s}$ and $\Lambda_{it}^s \equiv \left(\frac{\kappa_{it}^s A_{it}^s}{\xi_i^s}\right)^{\theta^s}$, the change in a competitiveness index is obtained as

$$d\ln S_{it} = d\ln\kappa_{it}^{\theta} - d\ln\tilde{P}_{it}^{\theta}, \tag{23}$$

where $d\ln\kappa_{it}^{\theta} \equiv \left(\theta^1 d\ln\kappa_{it}^1, ..., \theta^S d\ln\kappa_{it}^S\right)^T$ and $d\ln\tilde{P}_{it}^{\theta} \equiv \left(\theta^1 d\ln\tilde{P}_{it}^1, ..., \theta^S d\ln\tilde{P}_{it}^S\right)^T$. For tradable goods, we use exporter fixed effects $de\hat{x}_{it} = d\ln S_{it} - d\ln S_{bt}$ implying $\frac{1}{N}\sum_{i=1}^N de\hat{x}_{it} = \frac{1}{N}\sum_{i=1}^N d\ln S_{it} - d\ln S_{bt}$ and

$$\begin{aligned} d\ln S_{it} &= de\hat{x}_{it} - \frac{1}{N}\sum_{i=1}^N de\hat{x}_{it} + \frac{1}{N}\sum_{i=1}^N d\ln S_{it} \\ &= de\hat{x}_{it} - \frac{1}{N}\sum_{i=1}^N de\hat{x}_{it} - \frac{1}{N}\sum_{i=1}^N d\ln\tilde{P}_{it}^{\theta} \end{aligned}$$

from (23) and $\frac{1}{N}\sum_{i=1}^N d\ln\kappa_{it}^s = 0$. Combined shocks are obtained from (22) and (**??**). Quality shocks and productivity shocks are obtained from (23) and (**??**) as:

$$d\ln\kappa_{it}^{s}=\left(d\ln S_{it}^{s}+d\ln\tilde{P}_{it}^{s}\right)/\theta^{s}\text{ and }d\ln A_{it}^{s}=\left(d\ln\Lambda_{it}^{s}-d\ln S_{it}^{s}\right)/\theta^{s}-d\ln\tilde{P}_{it}^{s}.$$

For non-tradable service industries, we assume the relative quality across countries remains stable: $d\ln\kappa_{it}^{s}=0$, which implies from (23) $d\ln S_{it}^{s}=-\theta^{s}d\ln\tilde{P}_{it}^{s}$. Combined shocks and productivity shocks are obtained from (22) and $d\ln A_{it}^{s}=d\ln\Lambda_{it}^{s}/\theta^{s}$.

3.3 Estimation of Idiosyncratic Shocks

From the data of productivity and quality shocks estimated in the last section, we estimate the probability distribution of idiosyncratic productivity and quality shocks. Let $d\ln\tilde{A}_{it}^{s}\equiv d\ln A_{it}^{s}-\frac{1}{T}\sum_{t=1}^{T}d\ln A_{it}^{s}$ and $d\ln\tilde{\kappa}_{it}^{s}\equiv d\ln\kappa_{it}^{s}-\frac{1}{T}\sum_{t=1}^{T}d\ln\kappa_{it}^{s}$ be demeaned series. To decompose the shocks into common components and idiosyncratic components, we use the following three level factor model:

$$\begin{aligned}d\ln\tilde{A}_{it}^{s}&=\zeta_{is}^{gA}f_{t}^{gA}+\zeta_{is}^{cA}f_{it}^{cA}+\zeta_{is}^{sA}f_{st}^{sA}+\varepsilon_{ist}^{A}\\ d\ln\tilde{\kappa}_{it}^{s}&=\zeta_{is}^{g\kappa}f_{t}^{g\kappa}+\zeta_{is}^{c\kappa}f_{it}^{c\kappa}+\zeta_{is}^{s\kappa}f_{st}^{s\kappa}+\varepsilon_{ist}^{\kappa}\end{aligned}\tag{24}$$

where for each variable $v\in\{A,\kappa\}$, f_{t}^{gv}, f_{it}^{cv}, and f_{st}^{sv} represent a global-level factor, country-level factors and industry-level factors, respectively; ζ_{is}^{gv}, ζ_{it}^{cv}, and ζ_{st}^{sv} associated factor loadings; ε_{ist}^{v} idiosyncratic factors. Factors represent common shocks that affects each group at the same timing, while factor loadings capture the impact of common shocks that can vary across country and industries. A possible alternative method of extracting idiosyncratic shocks is a decomposition based on OLS with dummies (e.g., Koren and Tenreyro, 2007):

$$d\ln\tilde{A}_{it}^{s}=f_{it}^{A}+f_{st}^{A}+\varepsilon_{ist}^{A},$$

where f_{it}^{A} and f_{st}^{A} are estimated by country–year dummies and industry–year dummies, respectively. Compared to this dummy approach, our model (24) nests the above dummy model as a special cases: $\zeta_{is}^{gA}=0$ and $\zeta_{is}^{cA}=1$ and $\zeta_{is}^{sA}=1$, which means our model is more flexible.

Table 3: Variance Decomposition

Variable	Volatility	Variance Share			
		Global	Country	Industry	Idiosyncratic
Productivity	0.548	0.184	0.370	0.145	0.300
Quality	1.018	0.164	0.352	0.108	0.377
Lambda	0.768	0.359	0.582	0.255	0.594

Note: Volatility: the standard deviation. Third to sixth columns show the share of the variances of the factors times loadings of global-level, country-level, industry-level and idiosyncratic in the total variance.

We extract the three level factors sequentially, assuming factors are orthogonal across levels.[5] The first step extracts global factors. The second step extracts country-level factors from the first step residuals. The third step extracts industry-level factors from the second step residuals. The extraction of factors uses the principal components method (see, e.g., Bai and Ng, 2008).[6] Since the factors are orthogonal across levels, the variance of productivity shocks and quality shocks can be decomposed as the sum of variance at each level:

$$Var\left(d\ln\tilde{A}_{it}^{s}\right) = Var\left(\zeta_{is}^{gA}f_{t}^{gA}\right) + Var\left(\zeta_{is}^{cA}f_{it}^{cA}\right) + Var\left(\zeta_{is}^{sA}f_{st}^{sA}\right) + Var\left(\varepsilon_{ist}^{A}\right)$$
$$Var\left(d\ln\tilde{\kappa}_{it}^{s}\right) = Var\left(\zeta_{is}^{g\kappa}f_{t}^{g\kappa}\right) + Var\left(\zeta_{is}^{c\kappa}f_{it}^{c\kappa}\right) + Var\left(\zeta_{is}^{s\kappa}f_{st}^{s\kappa}\right) + Var\left(\varepsilon_{ist}^{\kappa}\right).$$

In the second column in Table 3, the first two rows show the standard deviation (volatility) of productivity shock $\theta^s d\ln\tilde{A}_{it}^{s}$ and quality shocks $\theta^s d\ln\tilde{\kappa}_{it}^{s}$ where θ^s is multiplied to be comparable with the combined shock (Lambda) in the third row. Quality is more volatile than productivity. Productivity and quality are negatively correlated with the correlation coefficient -0.66. Columns 3 to 6 show the share of the variances of the factors times loadings of global-level, country-level, industry-level and idiosyncratic in the total variance. All the four-level shocks are important sources of variances. Idiosyncratic shocks account for more than 30% of the variance of both productivity and quality shocks.

We estimate the probability distribution of idiosyncratic shocks as follows. Figure 1 plots the

[5] For instance, f_t^{gA} and f_{it}^{cA} are orthogonal, but f_{it}^{cA} and f_{jt}^{cA} may be correlated. Also, a factor of productivity may be correlated with that of quality.

[6] In theory, the model (24) may include more than one factor at each level and decide the number of factors based on statistical tests. However, with relatively short panel date $T = 14$, those tests requiring a large sample are not expected to have a satisfactory power. We plan to conduct a robustness check by changing the number of factors at each level to see how the results change.

density of the standard normal distribution and the histograms of idiosyncratic shocks of productivity (left) and quality (right) that are divided by the sample standard deviations calculated at the country-industry level. The two histograms are both very close to the normal distribution. The Kolmogorov-Smirnov test does not reject a null hypothesis that idiosyncratic shocks follow the normal distribution.[7] From these results, we assume ε^A_{ist} and ε^κ_{ist} follow independent normal distributions with mean 0 and country-industry specific variances $\left(\sigma^A_{is}\right)^2$ and $(\sigma^\kappa_{is})^2$, respectively, where variances are estimated by the sample variances at the country-industry level.

With the estimated distribution of idiosyncratic shocks, we obtain R samples of the combined shocks $\left\{\left(\hat{\Lambda}^s_{it}(r)\right)^N_{i=1}\right\}^R_{i=1}$ as follows. First, we draw $R = 100$ samples of $\varepsilon^A_{ist}(r)$ and $\varepsilon^\kappa_{ist}(r)$ from the estimated distributions. Plugging them into the estimated factor model (24) and adding back the means of productivity and quality changes, we obtain counterfactual growth rates, $d\ln A^s_{it}(r)$, $d\ln\kappa^s_{it}(r)$ and $d\ln\Lambda^s_{it}(r) = \theta^s(d\ln\kappa^s_{it}(r) + d\ln A^s_{it}(r))$. Since $d\ln\Lambda^s_{it}(r) = \ln\Lambda^s_{it}(r) - \ln\Lambda^s_{it-1}$ and $d\ln\Lambda^s_{it} = \ln\Lambda^s_{it} - \ln\Lambda^s_{it-1}$, we obtain $\hat{\Lambda}^s_{it}(r) \equiv \Lambda^s_{it}(r)/\Lambda^s_{it} = \exp\left(d\ln\Lambda^s_{it}(r) - d\ln\Lambda^s_{it}\right)$. Then, we obtain R samples of the combined shocks $\left\{\left(\hat{\Lambda}^s_{it}(r)\right)^N_{i=1}\right\}^R_{i=1}$.

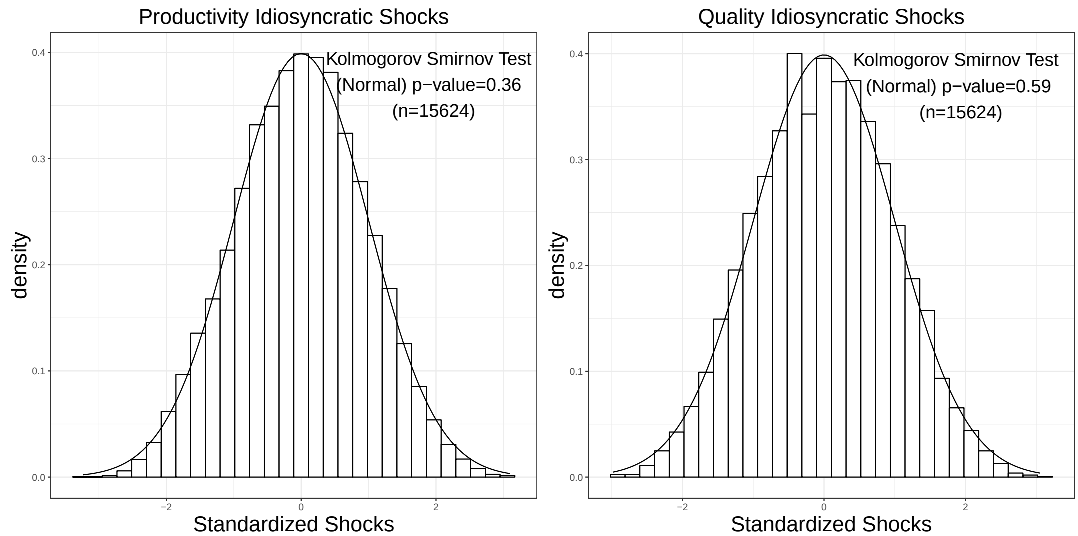
4 Counterfactual Analysis

4.1 Model Evaluation

The current model is developed to predict per capita income changes in counterfactual equilibriums. In this section, we evaluate the model's ability to predict per capita income changes. Using parameters estimated in the last section, the model can predict year-to-year changes in per capita income, i.e. wage w_{it}/w_{it-1}, for each country-year combination. The left-top panel in Figure 2 compares the model's prediction and data where the dashed line is the OLS fit. The correlation of prediction and data is 0.75 and the OLS fit line is very close to the 45 degree line, which supports the model's prediction performance. Compared to standard Ricardian models, the current model has added quality differentiation and distinction of usages. The other three panels examine the contribution of two new elements for the model's prediction ability. The goodness of fit of wage growth is shown for a model without quality differentiation in the top right panel, a model without

[7] When the variances are modeled as country-specific or industry-specific instead of country-industry specific, the Kolmogorov-Smirnov rejected the null hypothesis.

Figure 1: Distributions of Idiosyncratic Shocks



Note: Each figure plots the density of the standard normal distribution and the histograms of idiosyncratic shocks of productivity (left) and quality (right) that are normalized by the sample standard deviations calculated at the country-industry level. The Kolmogorov-Smirnov test does not reject a null hypothesis that idiosyncratic shocks follow the normal distribution.

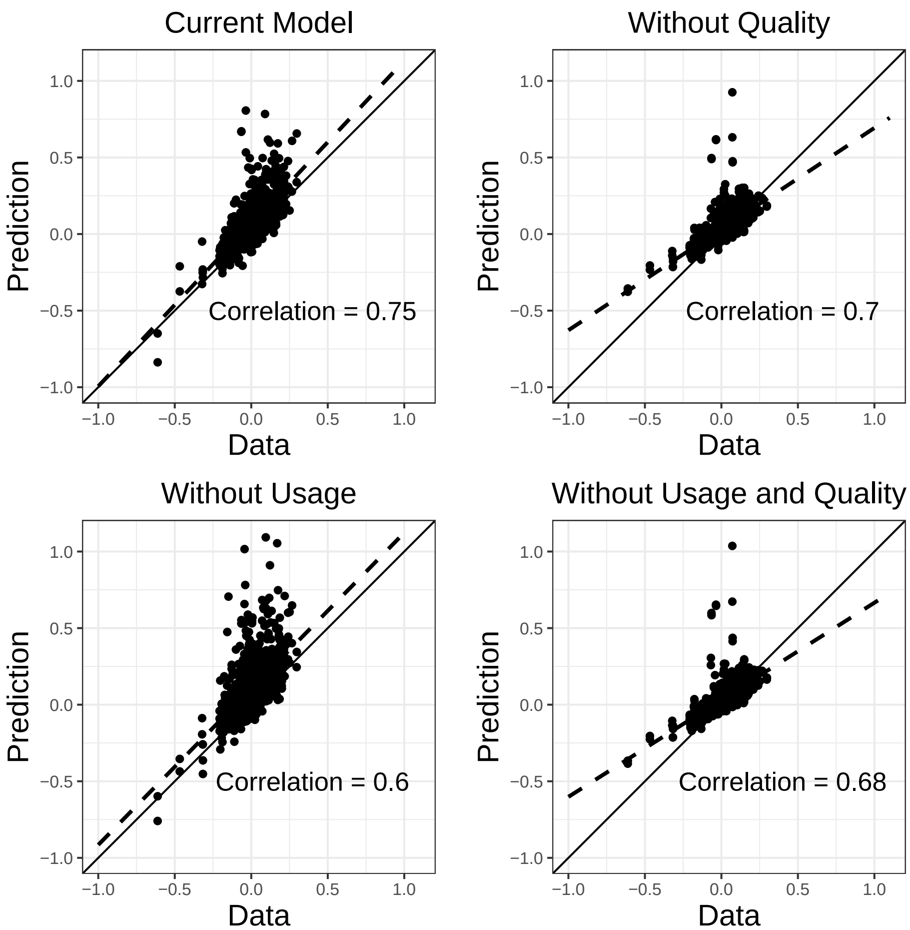
usage distinction in the bottom left panel and a model without both of them in the bottom right panel. A comparison of the panels shows the new elements improve the model's ability to predict per capita income changes.

4.2 GVC Drivers

The model endogenously predicts a pattern of global input-output linkages from trade costs, labor endowment and technology. This section identifies which of these three factors was the main determinant of the past rise in GVC. The literature has developed several measures about how deeply the GVC integrated the world production. Johnson (2018) provides an excellent review on those measures. We consider the share of foreign value added embodied in domestic production of final tradable goods, which we call FVA shares, developed by Timmer, Erumban, Los, Stehrer, and de Vries (2014) and Los, Timmer, and de Vries (2015). Since FVA shares are calculated from world-level input-output tables and the model predicts a world-level input-output table, we derive the model's prediction on FVA shares under counterfactual scenarios.[8] Appendix explains the details of the calculation.

[8] The entry regarding good s that industry r in country n's purchase from country s is $\beta_n^{rs}\pi_{nit}^s$.

Figure 2: Per Capital Income Growth: Models and Data



Note: Dashed lines are OLS fits.

The top left panel in Figure 3 compares the actual FVA shares of countries in 1995 and 2007. The dashed line is the OLS fit. As was documented by Timmer et al. (2014), most countries increase FVA shares between 1995 and 2007, which implies the GVC increased the integration of production during the period.

We detect the main driver of the GVC deepening as follows. We calculate counterfactual FVA shares by letting the three determinants (technology, endowment and trade costs) at the 1995 level one by one. If a chosen determinant is the main driver of GVC, then counterfactual FVA share in 2007 should move close to its 1995 level.

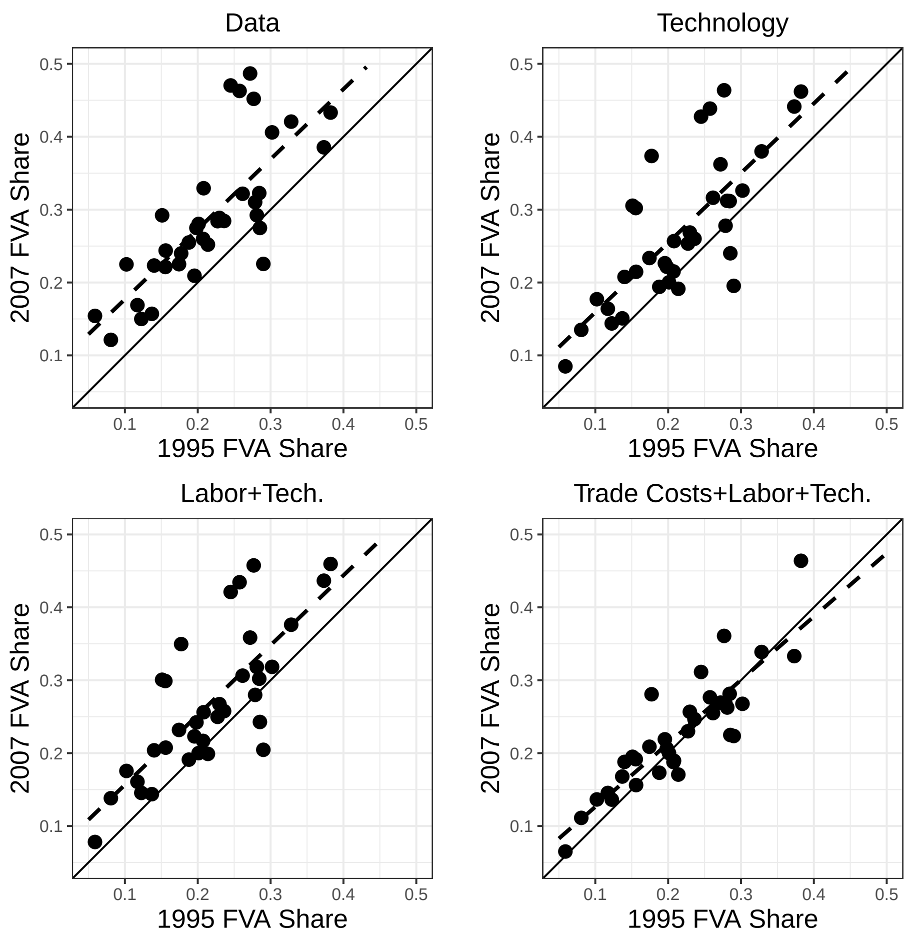
Figure 3 shows the result of this exercise. The top-right panel plots counterfactual FVA shares in 2007 under 1995 technology against actual FVA shares in 1995. Points and the OLS fit are still distant from the 45 degree line which implies that in the view of the current Ricardian model, technology is not likely to be the major driver of the GVC deepening. The bottom-left panel plots counterfactual FVA shares in 2007 when both labor endowment and technology are at the 1995 levels. The figure is still similar to that in the top right panel. The bottom right panel plots counterfactual FVA shares in 2007 when all the three determinants are at the 1995 levels. Points and the OLS fit line become very close to the 45 degree line.[9] From these results, we conclude that trade costs is the main driver of GVC in the view of the current Ricardian model.

4.3 First and Second Moment Impacts of GVC on Real Wage

Given that trade cost is the main driver of the GVC, we consider three different counterfactual scenarios on trade costs in the structural difference in difference analysis (15): (i) (Trade Costs 1995) trade costs are at the 1995 level; (ii) (No GVC) trade costs of intermediate goods are infinite; (iii) (Autarky) all trade costs are infinite. We also consider another scenario (iv) (No Final Trade): trade costs of final goods are infinite so that we compare gains from trade in final goods and in intermediate goods. For each scenario, we calculated $R = 100$ counterfactual equilibriums using the simulated sample of the combined shocks $\left\{\left(\hat{\Lambda}_{it}^{s}(r)\right)_{i=1}^{N}\right\}_{i=1}^{R}$. For the case of $d = 1$, we calculated $R = 100$ counterfactual equilibriums using the simulated sample of the combined shocks without changing trade costs. Then, we calculate ΔMW_{it} and ΔVW_{it} for each country and ΔMW_{wt} and ΔVW_{wt} for the world.

[9] We obtain a similar picture when only trade costs are kept at the 1995.

Figure 3: GVC Determinants



Note: Dashed lines are OLS fits.

4.3.1 Overall Effects

Panel A in Table 4 shows the effects on the world real wage in 2007. Since tariff revenues offset each other at the world level, the world real wage can be regarded as the world per capita real income. The first row shows that under all the four counterfactual scenarios, the world as a whole receives the first moment income gain. The world real wage increased by 6.5% relative to autarky and by 2.2% relative to the 1995 trade cost case . A comparison of the no GVC and autarky cases show that GVC by itself brought more than the half of total gains from trade. The no final trade case shows that GVC brought greater gains than trade in final goods. The second row reports the estimated volatility (standard deviation) of the world real wage under no trade cost change, which is the case of "with GVC" $d = 1$. Idiosyncratic shocks generate a moderate level of the world real wage volatility by 1.7%. The third row shows the change in the world real wage volatility due the GVC under different scenarios. The GVC increases volatility by only negligible rates. These results from Panel A suggest that the GVC increased the world real income without increasing the aggregate volatility at the world level.

Panel B shows the effects on individual countries. The first two rows report the mean and standard error of mean real wage changes in 2007 relative to different counterfactuals.[10] As in the case of the world-level effect, similar three patterns hold, which is reasonable because the world real wage is the population weighted average of countries' wages. First, on average, countries receive the first moment income gains under all scenarios. Second, GVC brought more than the half of total gains from trade for an average country. Third, GVC brought greater income gains than trade in final goods. A notable difference from the world-level case is that the first moment gain is greater at the country-level in Panel B than at the world-level result in Panel A. This suggests that countries with smaller population receive greater gains from the GVC.

The second set of two rows in Panel B report the mean and standard deviation (not standard errors) of estimated volatility of real wage under the current world with GVC. The real wage volatility generated by idiosyncratic shocks is roughly 0.035 with standard deviation 0.010. The fact that mean country-level volatility in Panel B is grater than the world-level volatility in Panel A suggests that idiosyncratic shocks create greater volatility for countries with small population.

[10] In scenarios (iii) and (iv), two countries are dropped because they do not produce any final goods.

Table 4: First and Second Moment Impacts of GVC on Real Wage

Panel A: World-level Effects

	Counterfactual Scenarios			
	Trade Costs 1995	No GVC	Autarky	No Final Trade
Mean World Real Wage Change in 2007	0.0224	0.0457	0.0653	0.0274
World Real Wage Volatility in 2007 with GVC	0.01719	0.01719	0.01720	0.01720
World Real Wage Volatility Change in 2007	0.00022	0.00025	0.00015	0.00038
Volatility Change (%)	1.3%	1.5%	0.9%	2.2%

Panel B: Country-level Effects

		Counterfactual Scenarios			
		Trade Costs 1995	No GVC	Autarky	No Final Trade
Mean Real Wage Change in 2007	Mean	0.0385	0.0849	0.1475	0.0697
	SE	(0.0060)	(0.0078)	(0.0141)	(0.0073)
Real Wage Volatility in 2007 with GVC	Mean	0.03545		0.03447	
	SD	0.01060		0.01003	
Real Wage Volatility Change in 2007	Mean	0.00080	0.00384	0.00409	0.00299
	SE	(0.00047)	(0.00100)	(0.00111)	(0.00065)
Volatility Change (%)	Mean	2.4%	11.7%	13.9%	9.3%
Number of Countries		35	35	33	33

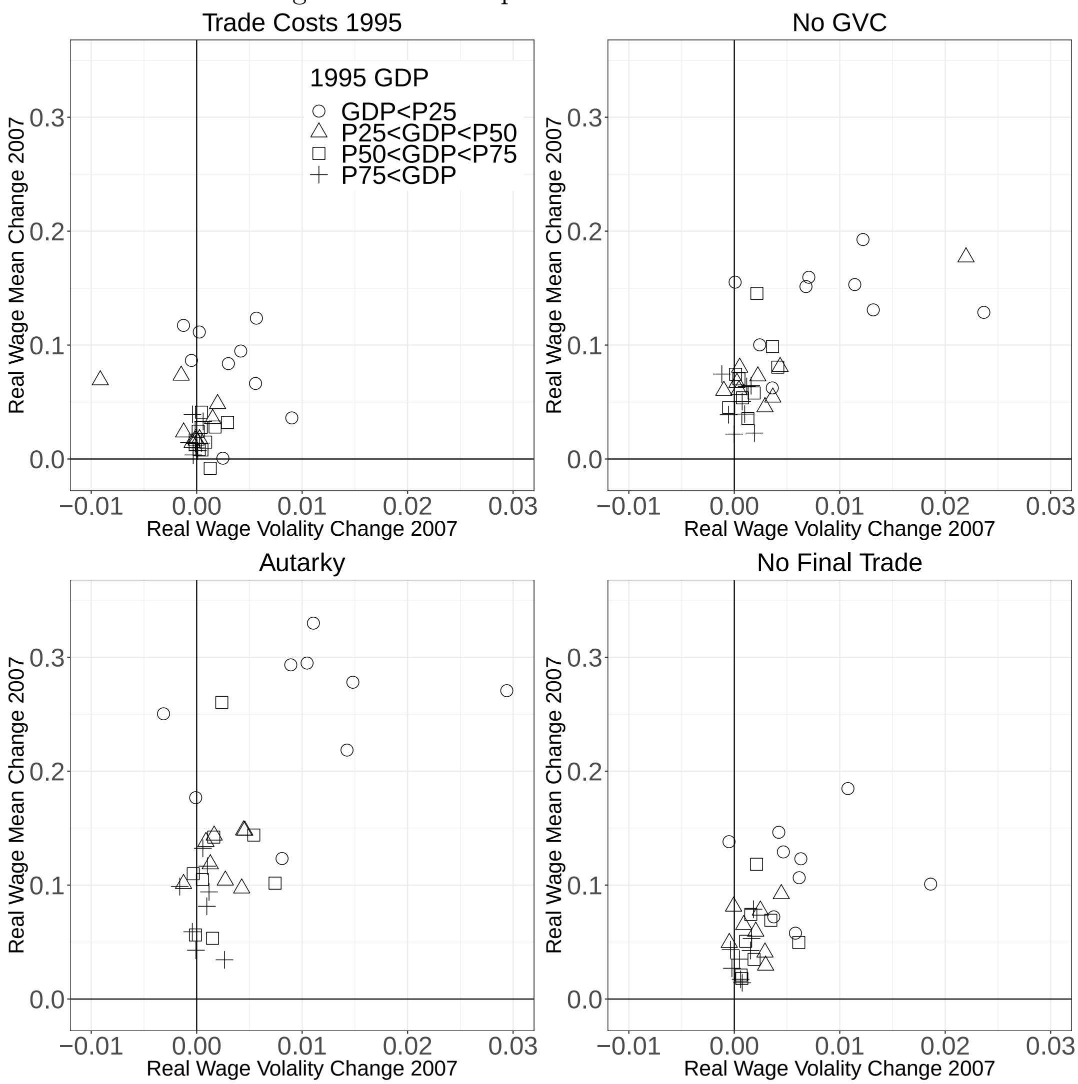
Note: SE: standard errors for mean. SD: standard deviation.

The third set of two rows report that the mean and standard error of real wage volatility changes in 2007 relative to different counterfactuals. The volatility changes are all positive, but the size largely varies across scenarios. In the case of 1995 trade costs, the volatility of an average country increases only by 2.4%, which is not statistically significant, while the increases in other cases are statistically significant. The volatility increased by 11.7% relative to the no GVC case and by 13.9% relative to autarky. A comparison of volatility changes between Panel A and Panel B shows that the volatility generated by the GVC is diversified at the world level but not at the individual country level.

4.3.2 Role of Country Size

The impact of the GVC on the mean and volatility of income is heterogeneous across countries, depending on the size of economies. Figure 4 plots the change in mean real wage against the change in real wage volatility of individual countries under the four scenarios. Different shapes of points represent different quartiles of initial GDP in 1995. The figure clearly shows that countries with

Figure 4: GVC Impacts and Economic Size



initially small GDP experienced the largest increases in both the mean and volatility of real wage.

Table 5 shows the regressions of ΔMW_{i2007} in Panel A and ΔVW_{i2007} in Panel B on country's economic size in 1995 such as GDP, employment and GDP per worker, respectively. In both Panel A and Panel B, both ΔMW_{i2007} and ΔVW_{i2007} are negatively correlated with 1995 GDP, 1995 employment and GDP per worker. The regression confirms the pattern in Figure 4. Initially small countries in terms of population or per capita income experienced the largest increases in both the mean and volatility of real wage.

In a future version of the current paper, we will investigate the mechanism behind these results.

Table 5: GVC Impacts and Economic Size

Panel A: First Moment Effects

	Mean Log Real Wage Change in 2007							
	1995 Trade Costs		No GVC		Autarky		No Final Trade	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
ln GDP 1995	-0.014***		-0.020***		-0.037***		-0.017***	
	(0.003)		(0.003)		(0.005)		(0.003)	
ln per capita		-0.012***		-0.020***		-0.039***		-0.020***
GDP 1995		(0.003)		(0.004)		(0.005)		(0.003)
ln Employment		-0.018***		-0.019***		-0.032***		-0.013***
1995		(0.004)		(0.005)		(0.007)		(0.004)
Constant	0.210***	0.205***	0.324***	0.324***	0.598***	0.605***	0.283***	0.289***
	(0.031)	(0.031)	(0.038)	(0.039)	(0.057)	(0.058)	(0.034)	(0.034)
Observations	35	35	35	35	33	33	33	33
R2	0.479	0.506	0.549	0.549	0.67	0.68	0.562	0.592

*p<0.10, **p<0.05, ***p<0.01

Panel B: Second Moment Effects

	Log Real Wage Volatility Change in 2007							
	1995 Trade Costs		No GVC		Autarky		No Final Trade	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
ln GDP 1995	-0.00073***		-0.00180***		-0.00211***		-0.00134***	
	(0.00025)		(0.00050)		(0.00051)		(0.00029)	
ln Employment		-0.00062**		-0.00188***		-0.00222***		-0.00136***
1995		(0.00028)		(0.00057)		(0.00058)		(0.00033)
ln Per capita		-0.00097**		-0.00163**		-0.00188**		-0.00131***
GDP 1995		(0.00036)		(0.00074)		(0.00076)		(0.00043)
Constant	0.00973***	0.00939***	0.02584***	0.02609***	0.02978***	0.03012***	0.01935***	0.01940***
	(0.00305)	(0.00309)	(0.00615)	(0.00629)	(0.00632)	(0.00645)	(0.00354)	(0.00363)
Observations	35	35	35	35	33	33	33	33
R2	0.20923	0.22841	0.28333	0.28553	0.35271	0.35644	0.41217	0.41243

*p<0.10, **p<0.05, ***p<0.01

Panel C: Summary Statistics of Variables in Regressions

Statistic	N	Mean	St. Dev.	Min	P25	Median	P75	Max
ln GDP 1995	35	12.223	1.752	8.173	11.296	12.486	13.247	15.82
ln Per Capita GDP 1995	35	2.939	1.241	-0.13	2.195	3.328	4.024	4.347
ln Employment 1995	35	9.284	1.625	6.449	8.225	9.158	10.118	13.457

5 (Tentative) Concluding Remarks

A potential increase in risks and uncertainty has been often mentioned as a negative major consequence of the globalization (e.g., Rodrik, 1997). This paper develops a framework to quantify the first and second moment effects of GVC on aggregate per capita income. Our tentative finding is that GVC increased the mean and volatility of the world real per capita income by 4.5% and 1.5%, respectively. At the individual country-level, GVC increased on average the mean and volatility of real per capita income by 8.5% and 11.7%, respectively, though the volatility disproportionately increases for poor and less populated countries. In sum, the GVC network aggregate idiosyncratic shocks to sizable country-level volatility but negligible world-level volatility. In a future version, we will investigate the mechanism behind our findings.

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A Online Appendix (Not for Publication)

A.1 Equilibrium Conditions in Matrix

A.1.1 Price-cost system (7) and (8)

Define $p_t^u(i,s) \equiv \ln\left(\hat{P}_{it}^{su*}\right)^{-\theta^s}$, $g_t^u(i,h,s) \equiv \pi_{iht}^{sm}\hat{\Lambda}_{ht}^{s}\hat{w}_{ht}^{-\theta^s\beta_h^s}\left(\hat{d}_{iht}^{sm}\right)^{-\theta^s}$ and $H_t(s,h) \equiv \sum_{k=1}^{S}\beta_h^{sk}p_t^m(h,k)$. Combining (7) and (8) by erasing c_{it}^s obtains

$$\exp\left[p_t^u(i,s)\right] = \sum_{h=1}^{N} g_t^m(i,h,s)\exp\left[H_t(s,h)\right].$$

It is written in the following matrix form:

$$\exp(p_t^u) = \left[G_t^u \circ (\iota_N \otimes \exp(H_t))\right]\iota_N \tag{25}$$

where $\iota_N \equiv (1,...,1)^T$ is $N \times 1$ vector, $\circ$ is an operator of element-by-element multiplication and

$$p_t^u(i) \equiv \begin{pmatrix} p_t^u(i,1) \\ p_t^u(i,2) \\ \vdots \\ p_t^u(i,S) \end{pmatrix} \text{ and } p_t^u \equiv \begin{pmatrix} p_t^u(1) \\ p_t^u(2) \\ \vdots \\ p_t^u(N) \end{pmatrix}$$

$$G_t^u(i,h) \equiv \begin{pmatrix} g_t^m(i,h,1) \\ g_t^m(i,h,2) \\ \vdots \\ g_t^m(i,h,S) \end{pmatrix} \text{ and } G_t^u \equiv \begin{pmatrix} G_t^u(1,1) & G_t^u(1,2) & \cdots & G_t^u(1,N) \\ G_t^u(2,1) & G_t^u(2,2) & \cdots & G_t^u(2,N) \\ \vdots & \vdots & \ddots & \vdots \\ G_t^u(N,1) & G_t^u(N,2) & \cdots & G_t^u(N,N) \end{pmatrix}$$

$$H_t(h) \equiv \begin{pmatrix} H_t(1,h) \\ H_t(2,h) \\ \vdots \\ H_t(S,h) \end{pmatrix} \text{ and } H_t \equiv \begin{pmatrix} H_t(1) & H_t(2) & \cdots & H_t(N) \end{pmatrix}.$$

G_t and H_t are obtained from data as follows.

G matrix Define $S \times 1$ vectors: $\pi^u_{ijt} \equiv \left(\pi^{1u}_{ijt}, ..., \pi^{Su}_{ijt}\right)^T$, $\hat{d}^u_{ijt} \equiv \left(\hat{d}^{1u}_{ijt}, ..., \hat{d}^{Su}_{ijt}\right)^T$, $\theta \equiv \left(\theta^1, ..., \theta^S\right)^T$, $\beta_i \equiv \left(\beta^1_i, ..., \beta^S_i\right)^T$, $\Lambda_{it} \equiv \left(\hat{\Lambda}^1_{it}, ..., \hat{\Lambda}^S_{it}\right)^T$ and $N \times 1$ vectors $\ln \hat{w}_t = (\ln \hat{w}_{1t}, ..., \ln \hat{w}_{1t})^T$. Stack these vectors to obtain $NS \times N$ matrices:

$$\Theta \equiv \iota_N \iota_N^T \otimes \theta,\ \Pi^u_t \equiv \begin{pmatrix} \pi^u_{11t} & \pi^u_{12t} & \cdots & \pi^u_{1Nt} \\ \pi^u_{21t} & \pi^u_{22t} & \cdots & \pi^u_{2Nt} \\ \vdots & \vdots & \ddots & \vdots \\ \pi^u_{N1t} & \pi^u_{N2t} & \cdots & \pi^u_{NNt} \end{pmatrix} \text{ and } D^u_t \equiv \begin{pmatrix} \hat{d}^u_{11t} & \hat{d}^u_{12t} & \cdots & \hat{d}^u_{1Nt} \\ \hat{d}^u_{21t} & \hat{d}^u_{22t} & \cdots & \hat{d}^u_{2Nt} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{d}^u_{N1t} & \hat{d}^u_{N2t} & \cdots & \hat{d}^u_{NNt} \end{pmatrix}.$$

Define $S \times N$ matrices: $\Lambda \equiv \begin{pmatrix} \Lambda_{1t} & \cdots & \Lambda_{Nt} \end{pmatrix}$, $\beta_l \equiv \begin{pmatrix} \beta_1 & \cdots & \beta_N \end{pmatrix}$ and

$$\exp\left[-\theta \left(\ln \hat{w}_t\right)^T \circ \beta_l\right] = \begin{pmatrix} \hat{w}_{1t}^{-\theta_1 \beta^1_1} & \hat{w}_{2t}^{-\theta_1 \beta^1_2} & \cdots & \hat{w}_{Nt}^{-\theta_1 \beta^1_N} \\ \hat{w}_{1t}^{-\theta_2 \beta^2_1} & \hat{w}_{2t}^{-\theta_2 \beta^2_2} & \cdots & \hat{w}_{Nt}^{-\theta_2 \beta^2_N} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{w}_{1t}^{\theta_S \beta^S_1} & \hat{w}_{2t}^{-\theta_S \beta^S_2} & \cdots & \hat{w}_{Nt}^{-\theta_S \beta^S_N} \end{pmatrix}.$$

Matrix G^u_t is obtained by

$$G^u_t = \Pi^u_t \circ \left(\iota_N \otimes \Lambda_t\right) \circ \exp\left[-\Theta \circ \ln D^u_t\right] \circ \left[\iota_N \otimes \exp\left[-\theta \left(\ln \hat{w}_t\right)^T \circ \beta_l\right]\right]. \tag{26}$$

H matrix Stack the transposes of IO matrices

$$\Gamma_{it} \equiv B^T_{it} = \begin{pmatrix} \beta^{11}_i & \beta^{12}_i & \cdots & \beta^{1S}_i \\ \beta^{21}_i & \beta^{22}_i & \cdots & \beta^{2S}_i \\ \vdots & \vdots & \ddots & \vdots \\ \beta^{S1}_i & \beta^{S2}_i & \cdots & \beta^{SS}_i \end{pmatrix} \text{ and } \Gamma_t \equiv \begin{pmatrix} \Gamma_{1t} & \Gamma_{2t} & \cdots & \Gamma_{Nt} \end{pmatrix}.$$

Since

$$H_t\left(i\right) = \begin{pmatrix} \sum_{k=1}^S \beta^{1k}_i p^m_t(i,k) \\ \sum_{k=1}^S \beta^{2k}_i p^m_t(i,k) \\ \vdots \\ \sum_{k=1}^S \beta^{Sk}_i p^m_t(i,k) \end{pmatrix} = \Gamma_{it} p^m_{it},$$

H is obtained as

$$
\begin{aligned}
H_t =& \begin{pmatrix} \Gamma_{1t}p_{1t}^m & \Gamma_{2t}p_{2t}^m & \cdots & \Gamma_{Nt}p_{Nt}^m \end{pmatrix} \\
=& \begin{pmatrix} \Gamma_{1t} & \Gamma_{2t} & \cdots & \Gamma_{Nt} \end{pmatrix} \begin{pmatrix} p_{1t}^m & 0 & \cdots & 0 \\ 0 & p_{2t}^m & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & p_{Nt}^m \end{pmatrix} \\
=& \Gamma_t \left(diag(p_t^m) \left(I_N \otimes \iota_S \right) \right),
\end{aligned}
\tag{27}
$$

where $diag(p_t^m)$ is a $NS \times NS$ matrix that places p_t^m in the diagonal elements and zeros in the off-diagonal elements (that is, its $((i-1)S+s)$th diagonal element includes $p_t^m(i,s)$).

A.1.2 Trade Shares (9)

The vector of the quality adjusted index is obtained as:

$$
\exp(p^f) = \left[G^f \circ \left(\iota_N \otimes \exp\left(H\right) \right) \right] \iota_N.
$$

From

$$
\exp(p^f\,(i,s)) = \sum_{h=1}^{N} \pi_{it}^{sf} \hat{\Lambda}_{ht}^s \left(\hat{d}_{iht}^{sf} \right)^{-\theta^s} \hat{w}_{ht}^{-\beta_h^s \theta^s} \exp\left[\sum_{k=1}^{S} \beta_h^{sk} p_t^m(h,k) \right],
$$

trade share (9) is written as

$$
\begin{aligned}
\pi_{iht}^{su\prime} &= \pi_{iht}^{su} \hat{\Lambda}_{ht}^s \left(\hat{c}_{ht}^s \hat{d}_{nht}^{su} \right)^{-\theta^s} \left(\hat{P}_{it}^{su*} \right)^{\theta^s}. \\
&= \pi_{iht}^{su} \hat{\Lambda}_{ht}^s \left(\hat{d}_{iht}^{sm} \right)^{-\theta^s} \hat{w}_{ht}^{-\theta^s \beta_h^s} \exp\left(\sum_{k=1}^{S} \beta_h^{sk} p_t^m(h,k) \right) \exp\left[-\ln\left(\hat{P}_{it}^{su*} \right)^{-\theta^s} \right] \\
&= g_t^u(i,h,s) \exp\left[H(s,h) \right] \exp\left[-p_t^u(i,s) \right].
\end{aligned}
$$

In matrix form, trade shares are obtained by

$$
\begin{aligned}
\Pi^u_{1t} &\equiv \begin{pmatrix} \pi^{u\prime}_{11t} & \pi^{u\prime}_{12t} & \cdots & \pi^{u\prime}_{1Nt} \\ \pi^{u\prime}_{21t} & \pi^{u\prime}_{22t} & \cdots & \pi^{u\prime}_{2Nt} \\ \vdots & \vdots & \ddots & \vdots \\ \pi^{u\prime}_{N1t} & \pi^{u\prime}_{N2t} & \cdots & \pi^{u\prime}_{NNt} \end{pmatrix} \\
&= G^u_t \circ \left(\iota_N \otimes \exp\left(H_t\right)\right) \circ \left[\exp(-p^u_t) \otimes \iota^T_N\right]. \qquad (28)
\end{aligned}
$$

A.1.3 Expenditure System (10) and (11)

Define

$$
a^{sku}_{nt} \equiv \alpha^s_n \sum_i \frac{\tau^{k\prime}_{nit}\pi^{ku\prime}_{nit}}{1+\tau^{k\prime}_{nit}} \text{ and } b^{sku}_{nit} \equiv \frac{\beta^{ks}_n \pi^{ku\prime}_{int}}{1+\tau^{k\prime}_{int}}.
$$

Equations (10) and (11) can be written as becomes

$$
\begin{aligned}
X^{sf\prime}_{nt} - \sum_{k=1}^{S} a^{skf}_{nt} X^{kf\prime}_{nt} - \sum_{k=1}^{S} a^{skm}_{nt} X^{km\prime}_{nt} &= \alpha^s_n \left[w^{\prime}_{nt} L_{nt} + D_{nt}\right] \\
X^{sm\prime}_{nt} - \sum_{i=1}^{N}\sum_{k=1}^{S} b^{skm}_{nit} X^{km\prime}_{it} - \sum_{i=1}^{N}\sum_{k=1}^{S} b^{skf}_{nit} X^{kf\prime}_{it} &= 0.
\end{aligned}
$$

The system is written in matrix forms:

$$
\begin{aligned}
(I - A^f_t) X^f_t - A^m_t X^m_t &= F_t \\
\left(I - B^m_t\right) X^m_t - B^f_t X^f_t &= 0
\end{aligned}
$$

where for $u = f, m$,

$$
X^u_{it} \equiv \begin{pmatrix} X^{1u\prime}_{it} \\ \vdots \\ X^{Su\prime}_{it} \end{pmatrix}, X^u_t \equiv \begin{pmatrix} X^u_{1t} \\ \vdots \\ X^u_{Nt} \end{pmatrix}
$$

$$A_{it}^{u} \equiv \begin{pmatrix} a_{it}^{11u} & a_{it}^{12u} & \cdots & a_{it}^{1Su} \\ a_{it}^{21u} & a_{it}^{22u} & \cdots & a_{it}^{2Su} \\ \vdots & \vdots & \ddots & \vdots \\ a_{it}^{S1u} & a_{it}^{S2u} & \cdots & a_{it}^{SSu} \end{pmatrix},\ A_{t}^{u} \equiv \begin{pmatrix} A_{1t}^{u} & 0 & 0 & 0 \\ 0 & A_{2t}^{u} & 0 & 0 \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & A_{Nt}^{u} \end{pmatrix}$$

$$B_{nit}^{u} \equiv \begin{pmatrix} b_{nit}^{11u} & b_{nit}^{12u} & \cdots & b_{nit}^{1Su} \\ b_{nit}^{21u} & b_{nit}^{22u} & \cdots & b_{nit}^{2Su} \\ \vdots & \vdots & \ddots & \vdots \\ b_{nit}^{S1u} & b_{nit}^{S2u} & \cdots & b_{nit}^{SSu} \end{pmatrix},\ B_{t}^{u} \equiv \begin{pmatrix} B_{11t}^{u} & B_{12t}^{u} & \cdots & B_{1Nt}^{u} \\ B_{21t}^{u} & B_{22t}^{u} & \cdots & B_{2Nt}^{u} \\ \vdots & \vdots & \ddots & \vdots \\ B_{N1t}^{u} & B_{N2t}^{u} & \cdots & B_{NNt}^{u} \end{pmatrix}$$

$$F_{it} \equiv \begin{pmatrix} \alpha_i^1 \left[\hat{w}_{it}\hat{L}_{it} w_{it} L_{it} + TD'_{it}\right] \\ \vdots \\ \alpha_i^S \left[\hat{w}_{it}\hat{L}_{it} w_{it} L_{it} + TD'_{it}\right] \end{pmatrix} = \text{ and } F_t \equiv \begin{pmatrix} F_{1t} \\ \vdots \\ F_{Nt} \end{pmatrix}.$$

A matrix, B matrix and F vectors are obtained as follows.

A matrix Define

$$T_{ijt}^{a} \equiv \begin{pmatrix} \frac{\tau_{ijt}^{1\prime}}{1+\tau_{ijt}^{1\prime}} \\ \vdots \\ \frac{\tau_{ijt}^{S\prime}}{1+\tau_{ijt}^{S\prime}} \end{pmatrix}, T_{t}^{a} \equiv \begin{pmatrix} T_{11t}^{a} & T_{12t}^{a} & \cdots & T_{1Nt}^{a} \\ T_{21t}^{a} & T_{22t}^{a} & \cdots & T_{2Nt}^{a} \\ \vdots & \vdots & \ddots & \vdots \\ T_{N1t}^{a} & T_{N2t}^{a} & \cdots & T_{NNt}^{a} \end{pmatrix},$$

$$\Psi_{it}^{u} \equiv \begin{pmatrix} \sum_j \frac{\tau_{ijt}^{1\prime}}{1+\tau_{ijt}^{1\prime}} \pi_{ijt}^{1u\prime} \\ \vdots \\ \sum_j \frac{\tau_{ijt}^{S\prime}}{1+\tau_{ijt}^{S\prime}} \pi_{ijt}^{Su\prime} \end{pmatrix} = \left(\pi_{ijt}^{u\prime} \circ T_{ijt}^{a}\right) \iota_N,\ \Psi_{t}^{u} \equiv \begin{pmatrix} \Psi_{1t}^{u} \\ \Psi_{2t}^{u} \\ \vdots \\ \Psi_{Nt}^{u} \end{pmatrix} = \left(\Pi_{1t}^{u} \circ T_{t}^{a}\right) \iota_N.$$

Since

$$\alpha_n \Psi_{nt}^T = \begin{pmatrix} \alpha_n^1 \sum_i \frac{\tau_{nit}^{1\prime}\pi_{nit}^{1\prime}}{1+\tau_{nit}^{1\prime}} & \alpha_n^1 \sum_i \frac{\tau_{nit}^{2\prime}\pi_{nit}^{2\prime}}{1+\tau_{nit}^{2\prime}} & \cdots & \alpha_n^1 \sum_i \frac{\tau_{nit}^{S\prime}\pi_{nit}^{S\prime}}{1+\tau_{nit}^{S\prime}} \\ \alpha_n^2 \sum_i \frac{\tau_{nit}^{1\prime}\pi_{nit}^{1\prime}}{1+\tau_{nit}^{1\prime}} & \alpha_n^2 \sum_i \frac{\tau_{nit}^{2\prime}\pi_{nit}^{2\prime}}{1+\tau_{nit}^{2\prime}} & \cdots & \alpha_n^2 \sum_i \frac{\tau_{nit}^{S\prime}\pi_{nit}^{S\prime}}{1+\tau_{nit}^{S\prime}} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_n^S \sum_i \frac{\tau_{nit}^{1\prime}\pi_{nit}^{1\prime}}{1+\tau_{nit}^{1\prime}} & \alpha_n^S \sum_i \frac{\tau_{nit}^{2\prime}\pi_{nit}^{2\prime}}{1+\tau_{nit}^{2\prime}} & \cdots & \alpha_n^S \sum_i \frac{\tau_{nit}^{S\prime}\pi_{nit}^{S\prime}}{1+\tau_{nit}^{S\prime}} \end{pmatrix}$$

$$= \begin{pmatrix} a_{it}^{11u} & a_{it}^{12u} & \cdots & a_{it}^{1Su} \\ a_{it}^{21u} & a_{it}^{22u} & \cdots & a_{it}^{2Su} \\ \vdots & \vdots & \ddots & \vdots \\ a_{it}^{S1u} & a_{it}^{S2u} & \cdots & a_{it}^{SSu} \end{pmatrix} = A_{nt}^u,$$

,

$$A_t^u = \begin{pmatrix} \alpha_1 \Psi_{1t}^{uT} & 0 & \cdots & 0 \\ 0 & \alpha_2 \Psi_{2t}^{uT} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \alpha_N \Psi_{Nt}^{uT} \end{pmatrix}$$

$$= \begin{pmatrix} \alpha_1 & 0 & \cdots & 0 \\ 0 & \alpha_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & \alpha_N \end{pmatrix} \begin{pmatrix} \Psi_{1t}^{uT} & 0 & \cdots & 0 \\ 0 & \Psi_{2t}^{uT} & \cdots & 0 \\ 0 & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Psi_{Nt}^{uT} \end{pmatrix}$$

$$= diag\left(\alpha\right)\left(I_N \otimes \iota_S\right)\left(I_N \otimes \iota_S^T\right) diag\left[\left(\Pi_t^u \circ T_t^a\right)\iota_N\right].$$

B matrix Define

$$T_{ijt}^{b} \equiv \begin{pmatrix} \frac{1}{1+\tau_{ijt}^{1\prime}} \\ \vdots \\ \frac{1}{1+\tau_{ijt}^{S\prime}} \end{pmatrix}, T_{t}^{b} \equiv \begin{pmatrix} T_{11t}^{b} & T_{12t}^{b} & \cdots & T_{1Nt}^{b} \\ T_{21t}^{b} & T_{22t}^{b} & \cdots & T_{2Nt}^{b} \\ \vdots & \vdots & \ddots & \vdots \\ T_{N1t}^{b} & T_{N2t}^{b} & \cdots & T_{NNt}^{b} \end{pmatrix}$$

$$\Phi_{ijt}^{u} \equiv \begin{pmatrix} \frac{\pi_{ijt}^{1u\prime}}{1+\tau_{ijt}^{1\prime}} \\ \vdots \\ \frac{\pi_{ijt}^{Su\prime}}{1+\tau_{ijt}^{S\prime}} \end{pmatrix} \text{ and } \Phi_{t}^{u} \equiv \begin{pmatrix} \Phi_{11t}^{u} & \Phi_{12t}^{u} & \cdots & \Phi_{1Nt}^{u} \\ \Phi_{21t}^{u} & \Phi_{22t}^{u} & \cdots & \Phi_{2Nt}^{u} \\ \vdots & \vdots & \ddots & \vdots \\ \Phi_{N1t}^{u} & \Phi_{N2t}^{u} & \cdots & \Phi_{NNt}^{u} \end{pmatrix} = T_{t}^{b} \circ \Pi_{1t}^{u}.$$

Since

$$\begin{aligned} B_{nit}^{uT} &= \begin{pmatrix} b_{nit}^{11u} & b_{nit}^{21u} & \cdots & b_{nit}^{S1u} \\ b_{nit}^{12u} & b_{nit}^{22u} & \cdots & b_{nit}^{S2u} \\ \vdots & \vdots & \ddots & \vdots \\ b_{nit}^{1Su} & b_{nit}^{2Su} & \cdots & b_{nit}^{SSu} \end{pmatrix} \\ &= \begin{pmatrix} \beta_{n}^{11} \frac{\pi_{int}^{1u\prime}}{1+\tau_{int}^{1\prime}} & \beta_{n}^{12} \frac{\pi_{int}^{1u\prime}}{1+\tau_{int}^{1\prime}} & \cdots & \beta_{n}^{1S} \frac{\pi_{int}^{1u\prime}}{1+\tau_{int}^{1\prime}} \\ \beta_{n}^{21} \frac{\pi_{int}^{2u\prime}}{1+\tau_{int}^{2\prime}} & \beta_{n}^{22} \frac{\pi_{int}^{2u\prime}}{1+\tau_{int}^{2\prime}} & \cdots & \beta_{n}^{2S} \frac{\pi_{int}^{2u\prime}}{1+\tau_{int}^{2\prime}} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{n}^{S1} \frac{\pi_{int}^{Su\prime}}{1+\tau_{int}^{S\prime}} & \beta_{n}^{S2} \frac{\pi_{int}^{Su\prime}}{1+\tau_{int}^{S\prime}} & \cdots & \beta_{n}^{SS} \frac{\pi_{int}^{Su\prime}}{1+\tau_{int}^{S\prime}} \end{pmatrix} \\ &= \Gamma_{nt} \circ \left(\iota_{S}^{T} \otimes \Phi_{int}^{u} \right), \end{aligned}$$

we obtain

$$
\begin{aligned}
B_t^{uT} &= \begin{pmatrix} B_{11t}^{uT} & B_{21t}^{uT} & \cdots & B_{N1t}^{uT} \\ B_{12t}^{uT} & B_{22t}^{uT} & \cdots & B_{N2t}^{uT} \\ \vdots & \vdots & \ddots & \vdots \\ B_{1Nt}^{uT} & B_{2Nt}^{uT} & \cdots & B_{NNt}^{uT} \end{pmatrix} \\
&= \begin{pmatrix} \Gamma_{1t} \circ \left(\iota_S^T \otimes \Phi_{11t}^u\right) & \Gamma_{2t} \circ \left(\iota_S^T \otimes \Phi_{12t}^u\right) & \cdots & \Gamma_{Nt} \circ \left(\iota_S^T \otimes \Phi_{1Nt}^u\right) \\ \Gamma_{1t} \circ \left(\iota_S^T \otimes \Phi_{21t}^u\right) & \Gamma_{2t} \circ \left(\iota_S^T \otimes \Phi_{22t}^u\right) & \cdots & \Gamma_{Nt} \circ \left(\iota_S^T \otimes \Phi_{2Nt}^u\right) \\ \vdots & \vdots & \ddots & \vdots \\ \Gamma_{1t} \circ \left(\iota_S^T \otimes \Phi_{N1t}^u\right) & \Gamma_{2t} \circ \left(\iota_S^T \otimes \Phi_{N2t}^u\right) & \cdots & \Gamma_{Nt} \circ \left(\iota_S^T \otimes \Phi_{NNt}^u\right) \end{pmatrix} \\
&= \begin{pmatrix} \Gamma_{1t} & \Gamma_{2t} & \cdots & \Gamma_{Nt} \\ \Gamma_{1t} & \Gamma_{2t} & \cdots & \Gamma_{Nt} \\ \vdots & \vdots & \ddots & \vdots \\ \Gamma_{1t} & \Gamma_{2t} & \cdots & \Gamma_{Nt} \end{pmatrix} \circ \begin{pmatrix} \iota_S^T \otimes \Phi_{11t}^u & \iota_S^T \otimes \Phi_{12t}^u & \cdots & \iota_S^T \otimes \Phi_{1Nt}^u \\ \iota_S^T \otimes \Phi_{21t}^u & \iota_S^T \otimes \Phi_{22t}^u & \cdots & \iota_S^T \otimes \Phi_{2Nt}^u \\ \vdots & \vdots & \ddots & \vdots \\ \iota_S^T \otimes \Phi_{N1t}^u & \iota_S^T \otimes \Phi_{N2t}^u & \cdots & \iota_S^T \otimes \Phi_{NNt}^u \end{pmatrix} \\
&= \left(\iota_N \otimes \Gamma_t\right) \circ \left(\Phi_t^u \otimes \iota_S^T\right).
\end{aligned}
$$

Thus, it holds that

$$
B_t^u = \left[\left(\iota_N \otimes \Gamma_t\right) \circ \left(\Phi_t^u \otimes \iota_S^T\right)\right]^T.
$$

F vector Since$F_{it} = \left(w_{it}' L_{it}' + D_{it}'\right) \alpha_1$,

$$
\begin{aligned}
F_t &\equiv \begin{pmatrix} \left(w_{1t}' L_{1t}' + T D_{1t}'\right) \alpha_1 \\ \vdots \\ \left(w_{Nt}' L_{Nt}' + T D_{Nt}'\right) \alpha_N \end{pmatrix} \\
&= \begin{pmatrix} \alpha_1 & 0 & 0 & 0 \\ 0 & \alpha_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \alpha_N \end{pmatrix} \begin{pmatrix} w_{1t}' L_{1t}' + T D_{1t}' \\ w_{2t}' L_{2t}' + T D_{2t}' \\ \vdots \\ w_{Nt}' L_{Nt}' + T D_{Nt}' \end{pmatrix} \\
&= diag\left[\alpha\right] \left(I_N \otimes \iota_S\right) V_t
\end{aligned}
$$

Matrix V_t is obtained as

$$
\begin{aligned}
V_t \equiv & \begin{pmatrix} \hat{w}_{1t}\hat{L}_{1t}w_{10}L_{10} + TD'_{1t} \\ \hat{w}_{2t}\hat{L}_{2t}w_{20}L_{20} + TD'_{2t} \\ \vdots \\ \hat{w}_{Nt}\hat{L}_{Nt}w_{N0}L_{N0} + TD'_{Nt} \end{pmatrix} \\
&= \hat{w}_t \circ \hat{L}_t \circ w_0 \circ L_0 + TD_t \\
&= \hat{w}_t \circ \hat{L}_t \circ w_0 \circ L_0 + \hat{v}^{world} TD_0,
\end{aligned}
$$

where $w_0 \equiv (w_{10}, ..., w_{N0})^T$, $L_0 \equiv (L_{10}, ..., L_{N0})^T$, $TD_t \equiv (TD'_{1t}, ..., TD'_{Nt})^T$ and $TD_0 \equiv (TD_{10}, ..., TD_{N0})^T$ are $N \times 1$ vectors, and $\hat{v}^{world} \equiv ((w')^T L')/\left((w_0)^T L_0\right)$ is the change in the world GDP (scalar). The last equation holds because

$$
((w')^T L)^{-1} TD = \left((w_0)^T L_0\right)^{-1} TD_0
$$

is imposed.

A.1.4 Trade balance (12)

Imports minus exports

$$
\begin{aligned}
TD'_{nt} &= \sum_{s=1}^{S}\sum_{i=1}^{N}\left(\frac{\pi_{nit}^{sf\prime}X_{nt}^{sf\prime} + \pi_{nit}^{sm\prime}X_{nt}^{sm\prime}}{1+\tau_{nit}^{s\prime}} - \frac{\pi_{int}^{sf\prime}X_{it}^{sf\prime} + \pi_{int}^{sm\prime}X_{it}^{sm\prime}}{1+\tau_{int}^{s\prime}}\right). \\
&= \Xi_n^{fT} X_{nt}^f + \Xi_n^{mT} X_{nt}^m - \Upsilon_{nt}^{fT} X_t^f - \Upsilon_{nt}^{mT} X_t^m,
\end{aligned}
$$

where

$$\Xi_n^u \equiv \begin{pmatrix} \sum_{i=1}^N \frac{\pi_{nit}^{1u\prime}}{1+\tau_{nit}^{1\prime}} \\ \sum_{i=1}^N \frac{\pi_{nit}^{1u\prime}}{1+\tau_{nit}^{1\prime}} \\ \vdots \\ \sum_{i=1}^N \frac{\pi_{nit}^{Su\prime}}{1+\tau_{nit}^{S\prime}} \end{pmatrix} \text{ and } \Upsilon_{nt}^u \equiv \begin{pmatrix} \frac{\pi_{1nt}^{1u\prime}}{1+\tau_{1nt}^{1\prime}} \\ \vdots \\ \frac{\pi_{1nt}^{Su\prime}}{1+\tau_{1nt}^{S\prime}} \\ \frac{\pi_{2nt}^{1u\prime}}{1+\tau_{2nt}^{1\prime}} \\ \vdots \\ \frac{\pi_{Nnt}^{Su\prime}}{1+\tau_{Nnt}^{S\prime}} \end{pmatrix}.$$

In matrix,

$$TD_t = \left(\Xi^f X_t^f + \Xi^m X_t^m\right) - \left(\Upsilon_t^f X_t^f + \Upsilon_t^m X_t^m\right)$$

where

$$\Xi_t^u \equiv \begin{pmatrix} \Xi_{1t}^{uT} & 0 & 0 & 0 \\ 0 & \Xi_{2t}^{uT} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \Xi_{nt}^{uT} \end{pmatrix}, \; \Upsilon^u \equiv \begin{pmatrix} \Upsilon_{1t}^{uT} \\ \Upsilon_{2t}^{uT} \\ \vdots \\ \Upsilon_{nt}^{uT} \end{pmatrix}.$$

Since

$$\left(T_t^b \circ \Pi_{1t}^u\right)\iota_N = \begin{pmatrix} \frac{\pi_{11t}^{1u\prime}}{1+\tau_{11t}^{1\prime}} & \frac{\pi_{12t}^{1u\prime}}{1+\tau_{12t}^{1\prime}} & \cdots & \frac{\pi_{1Nt}^{1u\prime}}{1+\tau_{1Nt}^{1\prime}} \\ \frac{\pi_{11t}^{2u\prime}}{1+\tau_{11t}^{2\prime}} & \frac{\pi_{12t}^{2u\prime}}{1+\tau_{12t}^{2\prime}} & \cdots & \frac{\pi_{1Nt}^{2u\prime}}{1+\tau_{1Nt}^{2\prime}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\pi_{11t}^{Su\prime}}{1+\tau_{11t}^{S\prime}} & \frac{\pi_{12t}^{Su\prime}}{1+\tau_{12t}^{S\prime}} & \cdots & \frac{\pi_{1Nt}^{Su\prime}}{1+\tau_{1Nt}^{S\prime}} \\ \frac{\pi_{21t}^{1u\prime}}{1+\tau_{21t}^{1\prime}} & \frac{\pi_{22t}^{1u\prime}}{1+\tau_{22t}^{1\prime}} & \cdots & \frac{\pi_{2Nt}^{1u\prime}}{1+\tau_{22t}^{1\prime}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\pi_{N1t}^{Su\prime}}{1+\tau_{N1t}^{S\prime}} & \frac{\pi_{N2t}^{Su\prime}}{1+\tau_{N2t}^{S\prime}} & \cdots & \frac{\pi_{NNt}^{Su\prime}}{1+\tau_{NNt}^{S\prime}} \end{pmatrix}\iota_N = \begin{pmatrix} \Xi_{1t}^u \\ \Xi_{2t}^u \\ \vdots \\ \Xi_{Nt}^u \end{pmatrix},$$

Ξ_t^u is obtained by

$$\Xi_t^u \equiv \begin{pmatrix} \Xi_{1t}^{uT} & 0 & 0 & 0 \\ 0 & \Xi_{2t}^{uT} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \Xi_{Nt}^{uT} \end{pmatrix}$$

$$= \left(I_N \otimes \iota_S^T\right) diag\left[vec\left(\Xi_{1t}^u, \Xi_{2t}^u, ..., \Xi_{Nt}^u\right)\right]$$

$$= \left(I_N \otimes \iota_S^T\right) diag\left[vec\left(\left(T_b \circ \Pi_1^u\right) \iota_N\right)\right].$$

Since

$$\Upsilon_{nt}^u \equiv \begin{pmatrix} \frac{\pi_{1nt}^{1u\prime}}{1+\tau_{1nt}^{1\prime}} \\ \vdots \\ \frac{\pi_{1nt}^{Su\prime}}{1+\tau_{1nt}^{S\prime}} \\ \frac{\pi_{2nt}^{1u\prime}}{1+\tau_{2nt}^{1\prime}} \\ \vdots \\ \frac{\pi_{Nnt}^{Su\prime}}{1+\tau_{Nnt}^{S\prime}} \end{pmatrix},$$

obtain

$$
\begin{aligned}
\Upsilon_t^u \equiv \begin{pmatrix} \Upsilon_{1t}^{uT} \\ \Upsilon_{2t}^{uT} \\ \vdots \\ \Upsilon_{nt}^{uT} \end{pmatrix} &= \begin{pmatrix} \frac{\pi_{11t}^{1u\prime}}{1+\tau_{11t}^{1\prime}} & \cdots & \frac{\pi_{11t}^{Su\prime}}{1+\tau_{11t}^{S\prime}} & \frac{\pi_{21t}^{1u\prime}}{1+\tau_{21t}^{1\prime}} & \cdots & \frac{\pi_{N1t}^{Su\prime}}{1+\tau_{N1t}^{S\prime}} \\ \frac{\pi_{12t}^{1u\prime}}{1+\tau_{12t}^{1\prime}} & \cdots & \frac{\pi_{12t}^{Su\prime}}{1+\tau_{12t}^{S\prime}} & \frac{\pi_{22t}^{1u\prime}}{1+\tau_{22t}^{1\prime}} & \cdots & \frac{\pi_{N2t}^{Su\prime}}{1+\tau_{N2t}^{S\prime}} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\pi_{1Nt}^{1u\prime}}{1+\tau_{1Nt}^{1\prime}} & \cdots & \frac{\pi_{1Nt}^{Su\prime}}{1+\tau_{1Nt}^{S\prime}} & \frac{\pi_{2Nt}^{1u\prime}}{1+\tau_{2Nt}^{1\prime}} & \cdots & \frac{\pi_{NNt}^{Su\prime}}{1+\tau_{NNt}^{S\prime}} \end{pmatrix} \\
&= \begin{pmatrix} \frac{\pi_{11t}^{1u\prime}}{1+\tau_{11t}^{1\prime}} & \frac{\pi_{12t}^{1u\prime}}{1+\tau_{12t}^{1\prime}} & \cdots & \frac{\pi_{1Nt}^{1u\prime}}{1+\tau_{1Nt}^{1\prime}} \\ \frac{\pi_{11t}^{2u\prime}}{1+\tau_{11t}^{2\prime}} & \frac{\pi_{12t}^{2u\prime}}{1+\tau_{12t}^{2\prime}} & \cdots & \frac{\pi_{1Nt}^{2u\prime}}{1+\tau_{1Nt}^{2\prime}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\pi_{11t}^{Su\prime}}{1+\tau_{11t}^{S\prime}} & \frac{\pi_{12t}^{Su\prime}}{1+\tau_{12t}^{S\prime}} & \cdots & \frac{\pi_{1Nt}^{Su\prime}}{1+\tau_{1Nt}^{S\prime}} \\ \frac{\pi_{21t}^{1u\prime}}{1+\tau_{21t}^{1\prime}} & \frac{\pi_{22t}^{1u\prime}}{1+\tau_{22t}^{1\prime}} & \cdots & \frac{\pi_{2Nt}^{1u\prime}}{1+\tau_{22t}^{1\prime}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\pi_{N1t}^{Su\prime}}{1+\tau_{N1t}^{S\prime}} & \frac{\pi_{N2t}^{Su\prime}}{1+\tau_{N2t}^{S\prime}} & \cdots & \frac{\pi_{NNt}^{Su\prime}}{1+\tau_{NNt}^{S\prime}} \end{pmatrix}^T \\
&= \left(T_t^b \circ \Pi_{1t}^u\right)^T.
\end{aligned}
$$

A.2 Numerical Solution Algorithm

We solve an equilibrium for each year t separately.

1. Guess a counterfactual wage change $\hat{w}_t = (\hat{w}_{1t}, ..., \hat{w}_{Nt})$ where $\hat{w}_{it} = w_{it}'/w_{it}$ for $i = 1, .., N-1$ and $\hat{w}_{Nt} = 1$, which is normalized from the Warlas's law.

2. For given $\hat{w}_t$, solve

$$\exp(p^m) = \left[G^m \circ (\iota_N \otimes \exp(H))\right] \iota_N$$

for p^m by iteration.

3. Obtain p^f and trade shares by

$$\exp(p^f) = \left[G^f \circ (\iota_N \otimes \exp(H))\right] \iota_N$$

4. Obtain the trade shares

$$\Pi_1^u = G^u \circ (\iota_N \otimes \exp(H)) \circ \left[\exp(-p^u) \otimes \iota_N^T\right]$$

5. Solve

$$(I - A_t^f)X_t^f - A_t^m X_t^m = F_t$$
$$(I - B_t^m)\, X_t^m - B_t^f X_t^f = 0.$$

for X_t^f and X_t^m.

6. Calculate trade deficit

$$TD_t = \left(\Xi^f X_t^f + \Xi^m X_t^m\right) - \left(\Upsilon_t^f X_t^f + \Upsilon_t^m X_t^m\right).$$

7. The target is to match trade deficit relative to the world GDP

$$\frac{TD'_{nt}}{\sum_{i=1}^N w'_{it} L'_{it}} - \frac{TD_{n0}}{\sum_{i=1}^N w_{i0} L_{i0}} = 0$$
$$\Leftrightarrow \Delta_{nt} \equiv TD_n - \left(\sum_{i=1}^N \left(\frac{w_{i0} L_{i0}}{\sum_{i=1}^N w_{i0} L_{i0}}\right) \hat{w}_{it}\right) TD_{n0} = 0$$

for $n = 1, ..., N - 1$.

8. The above procedure can be written as a procedure of solving a system of nonlinear equations $\Delta_{nt}(\hat{w}_t) = 0$. Many computing languages have solvers of a system of nonlinear equations. We used a quasi-Newton method (Broyden) and started with initial values $\hat{w}_{it} = 1$ for all i.

A.3 Calculating GVC Measures in Counterfactuals

A.3.1 World Input Out Table

We obtain the world input output table using the proportional assumption. The entry of the world input-output table regarding the purchase by sector r in country n from sector s in country i becomes $\pi_{nit}^{sm\prime} \beta_n^{rs}$ because of the following argument. Out of a one dollar revenue in sector r in country n, β_n^{rs}

dollar is spent on good s because of the Cobb-Douglass technology. Out of β_n^{rs} dollar, $\pi_{nit}^{sm\prime}$ is spent on goods from country i because country i's market share in goods s in country n is $\pi_{nit}^{sm\prime}$. The key assumption is that exporter's trade share is the same across purchasing industries within a given country. Although this is a restrictive assumption, most international input-out tables including WIOD are constructed with this assumption.

The world input output table is

$$Z_t \equiv \begin{pmatrix} Z_{11t} & Z_{12t} & \cdots & Z_{1Nt} \\ Z_{21t} & Z_{22t} & \cdots & Z_{2Nt} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{N1t} & Z_{N2t} & \cdots & Z_{NNt} \end{pmatrix}, \text{ where } Z_{int} \equiv \begin{pmatrix} \pi_{nit}^{1m\prime}\beta_n^{11} & \pi_{nit}^{1m\prime}\beta_n^{21} & \cdots & \pi_{nit}^{1m\prime}\beta_n^{S1} \\ \pi_{nit}^{2m\prime}\beta_n^{12} & \pi_{nit}^{2m\prime}\beta_n^{22} & \cdots & \pi_{nit}^{2m\prime}\beta_n^{S2} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{nit}^{Sm\prime}\beta_n^{1S} & \pi_{nit}^{Sm\prime}\beta_n^{2S} & \cdots & \pi_{nit}^{Sm\prime}\beta_n^{SS} \end{pmatrix}.$$

Matrix Z_t is calculated by matrix operations as follows. Since

$$\begin{aligned} Z_{in} &= \begin{pmatrix} \pi_{nit}^{1m\prime} & \pi_{nit}^{1m\prime} & \cdots & \pi_{nit}^{1m\prime} \\ \pi_{nit}^{2m\prime} & \pi_{nit}^{2m\prime} & \cdots & \pi_{nit}^{2m\prime} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{nit}^{Sm\prime} & \pi_{nit}^{Sm\prime} & \cdots & \pi_{nit}^{Sm\prime} \end{pmatrix} \circ \begin{pmatrix} \beta_n^{11} & \beta_n^{21} & \cdots & \beta_n^{S1} \\ \beta_n^{12} & \beta_n^{22} & \cdots & \beta_n^{S2} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_n^{1S} & \beta_n^{2S} & \cdots & \beta_n^{SS} \end{pmatrix} \\ &= \left(\pi_{nit}^{m\prime} \otimes \iota_S^T\right) \circ B_n. \end{aligned}$$

where $\pi_{nit}^{m\prime} \equiv \left(\pi_{nit}^{1m\prime}, ..., \pi_{nit}^{Sm\prime}\right)^T$ and B_n is country n's input output table. This is simplified as

$$\begin{aligned}
Z_t &= \begin{pmatrix} Z_{11t} & Z_{12t} & \cdots & Z_{1Nt} \\ Z_{21t} & Z_{22t} & \cdots & Z_{2Nt} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{N1t} & Z_{N2t} & \cdots & Z_{NNt} \end{pmatrix} \\
&= \begin{pmatrix} \pi_{11t}^{m\prime} \otimes \iota_S^T & \pi_{21t}^{m\prime} \otimes \iota_S^T & \cdots & \pi_{N1t}^{m\prime} \otimes \iota_S^T \\ \pi_{12t}^{m\prime} \otimes \iota_S^T & \pi_{22t}^{m\prime} \otimes \iota_S^T & \cdots & \pi_{N2t}^{m\prime} \otimes \iota_S^T \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{1Nt}^{m\prime} \otimes \iota_S^T & \pi_{2Nt}^{m\prime} \otimes \iota_S^T & \cdots & \pi_{NNt}^{m\prime} \otimes \iota_S^T \end{pmatrix} \circ \begin{pmatrix} B_1 & B_2 & \cdots & B_N \\ B_1 & B_2 & \cdots & B_N \\ \vdots & \vdots & \ddots & \vdots \\ B_1 & B_2 & \cdots & B_N \end{pmatrix} \\
&= \left[\tilde{\Pi}_1^m \otimes \iota_S^T\right] \circ (\iota_N \otimes B)
\end{aligned}$$

where

$$\tilde{\Pi}_1^m \equiv \begin{pmatrix} \pi_{11t}^{m\prime} & \pi_{21t}^{m\prime} & \cdots & \pi_{N1t}^{m\prime} \\ \pi_{12t}^{m\prime} & \pi_{22t}^{m\prime} & \cdots & \pi_{N2t}^{m\prime} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{1Nt}^{m\prime} & \pi_{2Nt}^{m\prime} & \cdots & \pi_{NNt}^{m\prime} \end{pmatrix}, B \equiv \begin{pmatrix} B_1 & \cdots & B_N \end{pmatrix}.$$

The value of gross output in sector s in country i is

$$Y_{nt}^{s\prime} = \sum_{i=1}^{N} \frac{\pi_{int}^{sf\prime}}{1 + \tau_{int}^{s\prime}} X_{it}^{sf\prime} + \sum_{i=1}^{N} \frac{\pi_{int}^{sm\prime}}{1 + \tau_{int}^{s\prime}} X_{it}^{sm\prime}$$

In vector form,

$$
\begin{aligned}
Y_{nt} \equiv \begin{pmatrix} Y_{nt}^{1\prime} \\ Y_{nt}^{2\prime} \\ \vdots \\ Y_{nt}^{S\prime} \end{pmatrix} &= \begin{pmatrix} \sum_{i=1} \frac{\pi_{int}^{1f\prime}}{1+\tau_{int}^{1\prime}} X_{it}^{1f\prime} + \sum_{i=1} \frac{\pi_{int}^{1m\prime}}{1+\tau_{int}^{1\prime}} X_{it}^{km\prime} \\ \sum_{i=1} \frac{\pi_{int}^{2f\prime}}{1+\tau_{int}^{2\prime}} X_{it}^{2f\prime} + \sum_{i=1} \frac{\pi_{int}^{2m\prime}}{1+\tau_{int}^{2\prime}} X_{it}^{2m\prime} \\ \vdots \\ \sum_{i=1} \frac{\pi_{int}^{Sf\prime}}{1+\tau_{int}^{S\prime}} X_{it}^{Sf\prime} + \sum_{i=1} \frac{\pi_{int}^{Sm\prime}}{1+\tau_{int}^{S\prime}} X_{it}^{Sm\prime} \end{pmatrix} \\
&= \sum_u \sum_i \begin{pmatrix} \frac{\pi_{int}^{1u\prime}}{1+\tau_{int}^{1\prime}} \\ \frac{\pi_{int}^{2u\prime}}{1+\tau_{int}^{2\prime}} \\ \vdots \\ \frac{\pi_{int}^{Su\prime}}{1+\tau_{int}^{2\prime}} \end{pmatrix} \circ \begin{pmatrix} X_{it}^{1u\prime} \\ X_{it}^{2u\prime} \\ \vdots \\ X_{it}^{Su\prime} \end{pmatrix} \\
&= \sum_u \sum_i \Phi_{int}^u \circ X_{it}^u.
\end{aligned}
$$

In matrix

$$
Y_t = \begin{pmatrix} Y_{1t} \\ Y_{2t} \\ \vdots \\ Y_{Nt} \end{pmatrix} = \left(M_t^m + M_t^f \right) \iota_N
$$

where

$$
\begin{aligned}
M_t^u \equiv \begin{pmatrix} \Phi_{11t}^u & \Phi_{21t}^u & \cdots & \Phi_{N1t}^u \\ \Phi_{12t}^u & \Phi_{22t}^u & \cdots & \Phi_{N2t}^u \\ \vdots & \vdots & \ddots & \vdots \\ \Phi_{1Nt}^u & \Phi_{2Nt}^u & \cdots & \Phi_{NNt}^u \end{pmatrix} &\circ \left(\iota_N \otimes \begin{pmatrix} X_{1t}^u & X_{2t}^u & \cdots & X_{Nt}^u \end{pmatrix} \right). \\
= \Phi_t^{uT} \circ \left(\iota_N \otimes \begin{pmatrix} X_{1t}^u & X_{2t}^u & \cdots & X_{Nt}^u \end{pmatrix} \right).
\end{aligned}
$$

Let f_{int}^s be the value of final goods in sector s shipped from country i to country j. Its vector and matrix forms are

$$f_{int} \equiv \begin{pmatrix} f_{int}^{1} \\ f_{int}^{2} \\ \vdots \\ f_{int}^{S} \end{pmatrix} = \begin{pmatrix} \pi_{nit}^{1f\prime} X_{nt}^{1f\prime} \\ \pi_{nit}^{2f\prime} X_{nt}^{2f\prime} \\ \vdots \\ \pi_{nit}^{Sf\prime} X_{nt}^{Sf\prime} \end{pmatrix}$$

and

$$\begin{aligned} f_t &= \begin{pmatrix} f_{11t} & f_{12t} & \cdots & f_{1Nt} \\ f_{21t} & f_{22t} & \cdots & f_{2Nt} \\ \vdots & \vdots & \ddots & \vdots \\ f_{N1t} & f_{N2t} & \cdots & f_{NNt} \end{pmatrix} \\ &= \begin{pmatrix} \pi_{11t}^{f\prime} & \pi_{21t}^{f\prime} & \cdots & \pi_{N1t}^{f\prime} \\ \pi_{12t}^{f\prime} & \pi_{22t}^{f\prime} & \cdots & \pi_{N2t}^{f\prime} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{1Nt}^{f\prime} & \pi_{2Nt}^{f\prime} & \cdots & \pi_{NNt}^{f\prime} \end{pmatrix} \circ \begin{pmatrix} X_{1t}^{f\prime} & X_{1t}^{f\prime} & \cdots & X_{1t}^{f\prime} \\ X_{2t}^{f\prime} & X_{2t}^{f\prime} & \cdots & X_{2t}^{f\prime} \\ \vdots & \vdots & \ddots & \vdots \\ X_{Nt}^{f\prime} & X_{Nt}^{f\prime} & \cdots & X_{Nt}^{f\prime} \end{pmatrix} \\ &= \tilde{\Pi}_{1t}^{f} \circ \left(\iota_N^T \otimes \begin{pmatrix} X_{Nt}^{f\prime} \\ X_{Nt}^{f\prime} \\ \vdots \\ X_{Nt}^{f\prime} \end{pmatrix} \right). \end{aligned}$$

The value added vector is constructed as

$$
\begin{aligned}
VA_t &= Y_t - diag(Y_t) Z_t^T \iota_N \\
&= Y_t - \begin{pmatrix} diag(Y_{1t}) & 0 & \cdots & 0 \\ 0 & diag(Y_{2t}) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & diag(Y_{Nt}) \end{pmatrix} \begin{pmatrix} Z_{11}^T & Z_{21}^T & \cdots & Z_{N1}^T \\ Z_{12}^T & Z_{22}^T & \cdots & Z_{N2}^T \\ \vdots & \vdots & \ddots & \vdots \\ Z_{1N}^T & Z_{2N}^T & \cdots & Z_{NN}^T \end{pmatrix} \iota_{NS} \\
&= Y_t - \begin{pmatrix} diag(Y_{1t}) Z_{11}^T & diag(Y_{1t}) Z_{21}^T & \cdots & diag(Y_{1t}) Z_{N1}^T \\ diag(Y_{2t}) Z_{12}^T & diag(Y_{2t}) Z_{22}^T & \cdots & diag(Y_{2t}) Z_{N2}^T \\ \vdots & \vdots & \ddots & \vdots \\ diag(Y_{Nt}) Z_{1N}^T & diag(Y_{Nt}) Z_{2N}^T & \cdots & diag(Y_{Nt}) Z_{NN}^T \end{pmatrix} \iota_{NS} \\
&= Y_t - \begin{pmatrix} \sum_i \sum_k \pi_{1it}^{km\prime} \beta_1^{1k} Y_{1t}^1 \\ \vdots \\ \sum_i \sum_k \pi_{1it}^{km\prime} \beta_1^{Sk} Y_{1t}^S \\ \sum_i \sum_k \pi_{2it}^{km\prime} \beta_2^{1k} Y_{2t}^1 \\ \vdots \\ \sum_i \sum_k \pi_{Nit}^{km\prime} \beta_N^{Sk} Y_{Nt}^S \end{pmatrix}
\end{aligned}
$$

since

$$
diag(Y_{nt}) Z_{in}^T = \begin{pmatrix} \pi_{nit}^{1m\prime} \beta_n^{11} Y_{nt}^1 & \pi_{nit}^{2m\prime} \beta_n^{12} Y_{nt}^1 & \cdots & \pi_{nit}^{Sm\prime} \beta_n^{1S} Y_{nt}^1 \\ \pi_{nit}^{1m\prime} \beta_n^{21} Y_{nt}^2 & \pi_{nit}^{2m\prime} \beta_n^{22} Y_{nt}^2 & \cdots & \pi_{nit}^{Sm\prime} \beta_n^{2S} Y_{nt}^2 \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{nit}^{1m\prime} \beta_n^{S1} Y_{nt}^S & \pi_{nit}^{2m\prime} \beta_n^{S2} Y_{nt}^S & \cdots & \pi_{nit}^{Sm\prime} \beta_n^{SS} Y_{nt}^S \end{pmatrix}.
$$

A.3.2 GVC measure

From the accounting identity, gross output equals intermediate demand plus final demand:

$$
Y_t = Z_t Y_t + f_t \iota_{NS}.
$$

This implies that gross output required to produce a final good vector $f_t \iota_{NS}$ is

$$(1 - Z_t)^{-1} f_t \iota_{NS}.$$

Analogously, gross output required to produce a final good vector f is $(1 - Z_t)^{-1} f$.

Since the value added content of gross output Y_t is $diag(VA_t)(diag(Y_t))^{-1} Y_t$ by construction, the value-added content of a final good vector f is

$$diag(VA_t)(diag(Y_t))^{-1}(1 - Z_t)^{-1} f.$$

Let g_{ni}^s be country i's value-added in industry s embodied in final good production in country n and $g_{ni} \equiv \left(g_{ni}^1, ..., g_{ni}^S\right)$ is a vector expression. The matrix expression is obtained from data as follows:

$$\begin{aligned}
g_t \equiv &\begin{pmatrix} g_{11t} & g_{21t} & \cdots & g_{N1t} \\ g_{12t} & g_{22t} & \cdots & g_{N2t} \\ \vdots & \vdots & \ddots & \vdots \\ g_{1Nt} & g_{2Nt} & \cdots & g_{NNt} \end{pmatrix} \\
&= diag(VA_t)(diag(Y_t))^{-1}(1 - Z_t)^{-1} \begin{pmatrix} \sum_j f_{1j} & 0 & \cdots & 0 \\ 0 & \sum_j f_{2j} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \sum_j f_{Nj} \end{pmatrix} \\
&= diag(VA_t)(diag(Y_t))^{-1}(1 - Z_t)^{-1} diag(f_t \iota_N) \left(I_N \otimes \iota_S\right).
\end{aligned}$$

The last expression holds since

$$
\begin{aligned}
&\begin{pmatrix} \sum_j f_{1j} & 0 & \cdots & 0 \\ 0 & \sum_j f_{2j} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \sum_j f_{Nj} \end{pmatrix} \\
=&\begin{pmatrix} \sum_j f_{1j}^1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \sum_j f_{1j}^S & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \sum_j f_{2j}^1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & \sum_j f_{Nj}^S \end{pmatrix} \begin{pmatrix} \iota_S & 0 & \cdots & 0 \\ 0 & \iota_S & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \iota_S \end{pmatrix} \\
=&diag(f_t \iota_N)\left(I_N \otimes \iota_S\right).
\end{aligned}
$$

Similarly, let g_{ni}^{Ms} be country i's value-added in industry s embodied in country n's final good production of tradable goods. Suppose the first s^M industries are tradable goods industries and the other industries are non-tradable goods industries. Let I_{s^M} be $s^M \times s^M$ identity matrix and φ^M be $S \times S$ matrix such that

$$
\varphi^M = \begin{pmatrix} I_{s^M} & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}
$$

so that for $x = (x_1, .., x_S)$, $\varphi^M x = (x_1, ..x_{s^M}, 0, ..., 0)$.

$$
\begin{aligned}
g_t^M \equiv & \begin{pmatrix} g_{11t}^M & g_{21t}^M & \cdots & g_{N1t}^M \\ g_{12t}^M & g_{22t}^M & \cdots & g_{N2t}^M \\ \vdots & \vdots & \ddots & \vdots \\ g_{1Nt}^M & g_{2Nt}^M & \cdots & g_{NNt}^M \end{pmatrix} \\
&= diag(VA_t)(diag(Y_t))^{-1}(1 - Z_t)^{-1} \begin{pmatrix} \varphi^M \sum_j f_{1j} & 0 & \cdots & 0 \\ 0 & \varphi^M \sum_j f_{2j} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \varphi^M \sum_j f_{Nj} \end{pmatrix} \\
&= diag(VA_t)(diag(Y_t))^{-1}(1 - Z_t)^{-1} diag(f_t \iota_N) \left(I_N \otimes \iota_{s^M} \right).
\end{aligned}
$$

The share of foreign value-added value-added embodied in country n's final good production of tradable goods is obtained by

$$1 - \frac{\iota_S^T g_{nnt}^M}{\sum_j \iota_S^T g_{njt}^M}.$$